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CPLOAS_2 User Manual

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Abstract

Weak link (WL)/strong link (SL) systems are important parts of the overall operational design of high-consequence systems. In such designs, the SL system is very robust and is intended to permit operation of the entire system under, and only under, intended conditions. In contrast, the WL system is intended to fail in a predictable and irreversible manner under accident conditions and render the entire system inoperable before an accidental operation of the SL system. The likelihood that the WL system will fail to deactivate the entire system before the SL system fails (i.e., degrades into a configuration that could allow an accidental operation of the entire system) is referred to as probability of loss of assured safety (PLOAS). This report describes the Fortran 90 program CPLOAS_2 that implements the following representations for PLOAS for situations in which both link physical properties and link failure properties are time-dependent: (i) failure of all SLs before failure of any WL, (ii) failure of any SL before failure of any WL, (iii) failure of all SLs before failure of all WLs, and (iv) failure of any SL before failure of all WLs. The effects of aleatory uncertainty and epistemic uncertainty in the definition and numerical evaluation of PLOAS can be included in the calculations performed by CPLOAS_2.

Keywords: Aleatory uncertainty, CPLOAS_2, Epistemic uncertainty, Probability of loss of assured safety, Strong link, Uncertainty analysis, Weak link

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1 INTRODUCTION

Weak link (WL)/strong link (SL) systems are important parts of the overall operational design of high-consequence systems [1-6]. In such designs, the SL system is very robust and is intended to permit operation of the entire system under, and only under, intended conditions (e.g., by transmitting a command to activate the system). In contrast, the WL system is intended to fail in a predictable and irreversible manner under accident conditions (e.g., in the event of a fire) and render the entire system inoperable before an accidental operation of the SL system.

The likelihood that the WL system will fail to deactivate the entire system before the SL system fails (i.e., degrades into a configuration that could allow an accidental operation of the entire system) is referred to as probability of loss of assured safety (PLOAS). The descriptor loss of assured safety (LOAS) is used because failure of the WL system places the entire system in a inoperable configuration while failure of the SL system, although undesirable, does not necessarily result in an unintended operation of the entire system. Thus, safety is “assured” by failure of the WL system.

The CPLOAS_2 program implements models for PLOAS under a variety of combinations of WLs and SLs and also a variety of definitions for PLOAS. The CPLOAS_2 program takes physical properties of a system (e.g., temperature, pressure, …) calculated by mechanistic models for accident conditions and then uses these properties and definitions of link failure properties in probabilistic calculations to determine PLOAS. At the user’s request, CPLOAS_2 can also estimate the distribution of time margin before PLOAS as well as the distribution of environmental margin (such as temperature and pressure).

The CPLOAS_2 program was developed to replace the CPLOAS program ([7], App. III) and extends the computational capabilities in CPLOAS in several ways. First, the presence of aleatory uncertainty in system properties (e.g., temperature, pressure, …) can now be incorporated into PLOAS, which is not possible with CPLOAS. Second, the failure values for individual links can now be time-dependent functions of system properties, which is also not possible with CPLOAS. In addition, the capability to incorporate epistemic uncertainty into PLOAS results has been enhanced.

Finally the time margin and environmental margin distribution calculation has been added. Time margin looks at the margin in time before PLOAS occurs. It estimates the difference in time between a weak link failure and strong link failure. The selection of weak link and strong link (last or first one to fail) depend on the circuit type considered (see Table 1 for all four cases considered). The environmental margin looks at the margin in property before PLOAS occurs and estimates the difference between the property of the strong link of interest fails and the property of this strong link when the weak link of interest fails, with strong link and weak link of interest selected as for time margin, based on circuit type considered.

The CPLOAS_2 program, which is written in Fortran 90, consists of a set of 7 source files (cploascdf.f90, Distributions.f90, Files_IO.f90, margins.f90, misc_math.f90, quadrature.f90, Sampling.f90) as well as a Readme.txt file listing the changes in each versions of CPLOAS_2. The executable reads the same property file (CPCDF.DAT) and failure file (CPCDF.TPF) as its predecessor CPLOAS, but the user options are defined differently as described in Sect. 2.

The source code has been successfully compiled for windows platform using Intel® Visual Fortran Compiler Professional Edition 11.1 and tested on windows 7 and windows server 2008R2.

The probabilistic models and associated numerical procedures implemented in CPLOAS_2 are described in Ref. [8]. It is assumed in this user manual that a potential user of CPLOAS_2 has acquired a familiarity with the models described in Ref. [8] before attempting to use this user manual to guide the performance of a CPLOAS_2 calculation.

The following topics are considered in this report: input files read by CPLOAS_2 (Sect. 2), output files generated by CPLOAS_2 (Sect. 3), some considerations in the use of sampling in CPLOAS_2 (Sect. 4), and test cases for CPLOAS_2 (Sect. 5).

2 INPUT FILES READ BY CPLOAS_2

In the following, input files are described in what the authors feel is the most natural order for understanding the calculations performed in CPLOAS_2. Thus, the file descriptions start with the definition of WLs and SLs (Sect. 2.1) and then progress through the definition of WL/SL configurations (Sect. 2.2) and the control of calculations (Sect. 2.3).

2.1 Definition of WLs and SLs

As described in Sect. 2 of Ref. [8], each individual WL or SL and its associated cumulative distribution function (CDF) for time of failure are defined based on the following assumed link properties for the time interval $t_{mn} \le t \le t_{mx}$:

$$\bar{p}(t) = \text{nondecreasing function defining nominal link property for } t_{mn} \le t \le t_{mx}, \tag{2.1}$$

$$\bar{q}(t) = \text{nonincreasing function defining nominal failure value for link property for } t_{mn} \le t \le t_{mx}, \tag{2.2}$$

$$d_\alpha(\alpha) = \text{density function for variable } \alpha \text{ used to characterize aleatory uncertainty in link property}, \tag{2.3}$$

$$d_\beta(\beta) = \text{density function for variable } \beta \text{ used to characterize aleatory uncertainty in link failure value}, \tag{2.4}$$

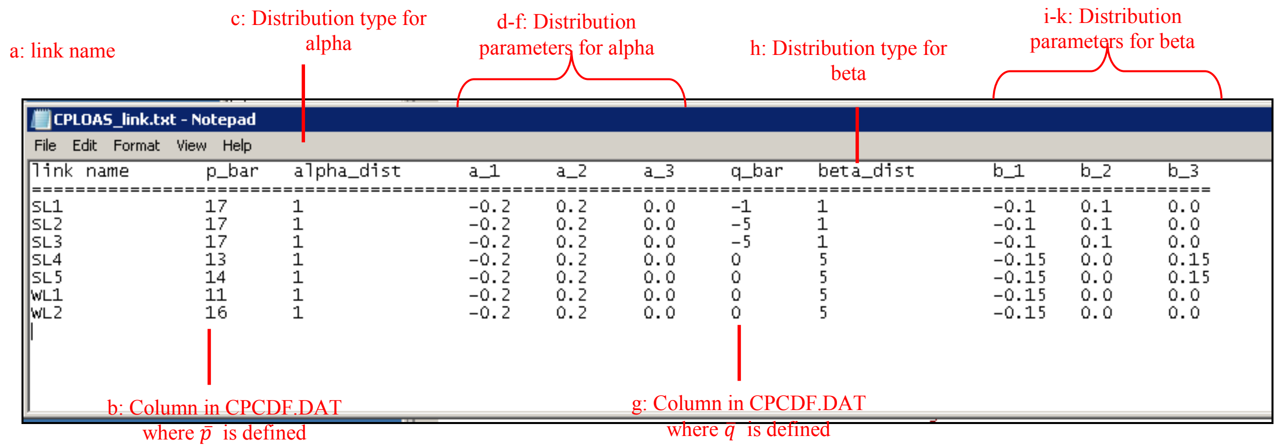
$$p(t\,|\,\alpha) = \alpha\,\bar{p}(t) = \text{ link property for } t_{mn} \le t \le t_{mx} \text{ given } \alpha, \tag{2.5}$$

and

$$q(t\,|\,\beta) = \beta\bar{q}(t) = \text{ link failure value for } t_{mn} \le t \le t_{mx} \text{ given } \beta. \tag{2.6}$$

Further, $d_\alpha(\alpha)$ and $d_\beta(\beta)$ are assumed to be defined on intervals $[\alpha_{mn}, \alpha_{mx}]$ and $[\beta_{mn}, \beta_{mx}]$ and to equal zero outside these intervals.

The link properties $\bar{p}(t)$ and $\bar{q}(t)$ indicated in Eqs. (2.1) and (2.2) and the distributions associated with the density functions $d_\alpha(\alpha)$ and $d_\beta(\beta)$ indicated in Eqs. (2.3) and (2.4) are input to CPLOAS 2 through the input file **CPLOAS_link.txt** illustrated in Fig. 1. The content of the individual columns in Fig. 1 is described below.



link name	p_bar	alpha_dist	a_1	a_2	a_3	q_bar	beta_dist	b_1	b_2	b_3
SL1	17	1	-0.2	0.2	0.0	-1	1	-0.1	0.1	0.0
SL2	17	1	-0.2	0.2	0.0	-5	1	-0.1	0.1	0.0
SL3	17	1	-0.2	0.2	0.0	-5	1	-0.1	0.1	0.0
SL4	13	1	-0.2	0.2	0.0	0	5	-0.15	0.0	0.15
SL5	14	1	-0.2	0.2	0.0	0	5	-0.15	0.0	0.15
WL1	11	1	-0.2	0.2	0.0	0	5	-0.15	0.0	0.0
WL2	16	1	-0.2	0.2	0.0	0	5	-0.15	0.0	0.0

Fig. 1 Example of input file **CPLOAS_link.txt** that defines WL and SL properties; content of Columns a-k defined in following text.

Fig. 1, Column a: Specifies names for the individual links, with each WL name starting with the letter "W" and each SL name starting with the letter "S" – **this requirement is mandatory**). Each row specifies properties of corresponding link named in this column.

Fig. 1, Column b: Specifies location for each link of information defining the nominal properties function $\bar{p}(t)$ indicated in Eq. (2.1). For each link, the corresponding positive integer in this column designates a column in the input file **CPCDF.DAT** illustrated in Fig. 2 that defines $\bar{p}(t)$. Specifically, the time column is not counted in this designation; thus a column designation of nC actually means that the definition of $\bar{p}(t)$ appears in column $nC + 1$ of **CPCDF.DAT**. The file **CPCDF.DAT** is structured as follows: (i) An initial row of comments naming each column and (ii) subsequent rows listing time-dependent properties (e.g., pressure and temperature) of the system under consideration, with the first column listing the times at which the properties are defined. Thus, the first column and any additional column in **CPCDF.DAT** define one time-dependent property of the system under consideration. Additional specification options for Column b include the use of 0 or a negative integer as described in conjunction with the description of Column g.

CPCDF.DAT - Notepad

File Edit Format View Help

# time	s1	c2	c3	c4	s5	c6	c7	c8	c9	c10
0.000	288.00000	288.00000	288.00000	288.00000	288.00000	288.00000	288.00000	288.00000	288.00000	288.00000
1.000	312.19650	310.35437	290.98019	290.69693	290.93344	290.89566	288.29633	288.29443	288.12805	288.14377
2.000	334.78491	331.39917	298.46967	297.61786	298.54178	298.39178	289.14246	289.13727	288.49637	288.56036
3.000	355.99515	351.32675	308.84412	307.37918	309.39572	309.07410	290.49942	290.49625	289.08420	289.23010
4.000	375.99530	370.27448	321.01144	318.99045	322.44370	321.91238	292.33649	292.34604	289.87411	290.13562
5.000	394.90988	388.34036	334.23291	331.74905	336.91306	336.15469	294.62317	294.66000	290.85092	291.26135
6.000	412.83234	405.59460	348.00677	345.15982	352.23697	351.25116	297.32880	297.41019	292.00128	292.59308
7.000	429.83417	422.08777	361.99161	358.87527	368.00031	366.79935	300.42285	300.56787	293.31311	294.11752
8.000	445.97144	437.85696	375.95502	372.65265	383.89975	382.50464	303.87537	304.10406	294.77530	295.82230
9.000	461.28943	452.93021	389.73926	386.32294	399.71457	398.15155	307.65714	307.98999	296.37756	297.69556
10.000	475.82599	467.32956	403.23792	399.76910	415.28491	413.58304	311.73990	312.19727	298.11029	299.72604
11.000	489.61401	481.07349	416.37997	412.91064	430.49619	428.68524	316.09647	316.69824	299.96432	301.90292
12.000	502.68314	494.17841	429.11911	425.69284	445.26736	443.37653	320.70071	321.46603	301.93100	304.21579
13.000	515.06097	506.65985	441.42618	438.07928	459.54245	457.59930	325.52768	326.47461	304.00211	306.65460
14.000	526.77399	518.53333	453.28427	450.04666	473.28430	471.31424	330.55359	331.69894	306.16983	309.20972
15.000	537.84802	529.81464	464.68515	461.58121	486.47000	484.49579	335.75580	337.11502	308.42667	311.87189
16.000	548.30865	540.52045	475.62686	472.67609	499.08734	497.12903	341.11285	342.69992	310.76550	314.63217
17.000	558.18134	550.66821	486.11212	483.32971	511.13242	509.20734	346.60449	348.43182	313.17957	317.48203
18.000	567.49164	560.27631	496.14703	493.54446	522.60767	520.73041	352.21161	354.29001	315.66238	320.41327
19.000	576.26501	569.36389	505.74033	503.32574	533.52039	531.70313	357.91626	360.25488	318.20789	323.41812
20.000	584.52698	577.95105	514.90277	512.68140	543.88177	542.13416	363.70160	366.30798	320.81027	326.48920
21.000	592.30292	586.05835	523.64661	521.62122	553.70587	552.03546	369.55188	372.43204	323.46408	329.61945
22.000	599.61798	593.70691	531.98529	530.15643	563.00903	561.42151	375.45245	378.61081	326.16412	332.80222
23.000	606.49689	600.91809	539.93304	538.29956	571.80920	570.30853	381.38971	384.82922	328.90561	336.03125
24.000	612.96399	607.71338	547.50488	546.06396	580.12573	578.71436	387.35107	391.07327	331.68396	339.30066
25.000	619.04297	614.11420	554.71613	553.46375	587.97864	586.65784	393.32492	397.32999	334.49490	342.60492
26.000	624.75696	620.14178	561.58240	560.51349	595.38867	594.15863	399.30057	403.58740	337.33447	345.93884
27.000	630.12830	625.81708	568.11938	567.22803	602.37677	601.23682	405.26822	409.83456	340.19894	349.29761
28.000	635.17841	631.16046	574.34277	573.62244	608.96405	607.91272	411.21893	416.06146	343.08484	352.67676
29.000	639.92792	636.19177	580.26807	579.71173	615.17139	614.20673	417.14459	422.25897	345.98895	356.07211
30.000	644.39655	640.93024	585.91052	585.51086	621.01941	620.13904	423.03778	428.41882	348.90833	359.47986

Fig. 2 Excerpt from input file **CPCDF.DAT** that defines time-dependent system properties; content of this file is described in conjunction with description of Column b of Fig. 1.

Fig. 1, Column c: Specifies distribution associated with density function $d_\alpha(\alpha)$ indicated in Eq. (2.3) that defines the aleatory uncertainty present in the function $p(t\,|\,\alpha) = \alpha\,\bar{p}(t)$ in Eq. (2.5). The integers 0, 1, 2, …, 6 appearing in this column designate the assigned distributions for α, which can also be assigned with the indicated code names. The following distributions are available for α values:

- 0, CST, C, DLT........... constant
- 1, UN, U, UNI............. Uniform distribution
- 2, LU, LGU .... Log uniform distribution
- 3, NO, N, NRM........... Normal distribution
- 4, LN, LGN .... Log normal distribution
- 5, T, TR, TRI .... Triangular distribution
- 6, LT, LGT .... Log-triangular distribution.

Fig. 1, Columns d-f : Specify defining parameters for distributions indicated in Column c. Columns d, f and g contain values for distribution parameters designated by a_1, a_2 and a_3, respectively, for each distribution:

(0) Constant: a_1 = constant value , a_2 =dummy parameter that is ignored after being read, and a_3 = dummy parameter that is ignored after being read;

(1) Uniform distribution: a_1 = minimum, a_2 = maximum, and a_3 = dummy parameter that is ignored after being read;

(2) Log uniform distribution: a_1 = minimum, a_2 = maximum, and a_3 = dummy parameter that is ignored after being read;

(3) Normal distribution: a_1 = mean, a_2 = standard deviation, and a_3 = quantile q expressed in decimal format used to truncate the distribution (i.e. the distribution will be truncated and normalized to a distribution defined between the q and $1-q$ quantiles of a normal distribution with parameters a_1 and a_2; see Ref. [8], Eqs. 3.14-3.17);

(4) Log normal distribution: a_1 = mean of ln(α), a_2 = standard deviation of ln(α), and a_3 = quantile q expressed in decimal format used to truncate the distribution (defined the same as for the normal distribution);

(5) Triangular distribution: a_1 = minimum, a_2 = mode (can be set to minimum or maximum), and a_3 = maximum;

(6) Log-triangular distribution: a_1 = minimum, a_2 = mode (can be set to minimum or maximum), and a_3 = maximum.

Fig. 1, Column g: Same as described for Column b but for the nominal failure value function $\bar{q}(t)$ indicated in Eq. (2.2) when a positive integer designating a column in the input file **CPCDF.DAT** is specified. Both Columns b and g also allow two additional specifications designated by zero or a negative integer. A zero indicates that the corresponding nominal value function (i.e., $\bar{p}(t)$ in Column b and $\bar{q}(t)$ in Column g) is to be assigned a constant value specified in the input file **CPCDF.TPF** illustrated in Fig. 3, and a negative integer (i.e., $-n$, where n is a column number in **CPCDF.DAT**) indicates that the time-dependent values for the nominal value function are to be initially read from Column n of **CPCDF.DAT** and then transformed to new values by a function defined in **CPCDF.TPF**. The file **CPCDF.TPF** is structured as follows: (i) Each link for which information is supplied is designated by a row that contains the number nR of rows of supplied information and the name of the link, (ii) if nR =1, then a single row of information is supplied and the nominal value function (i.e., $\bar{p}(t)$ or $\bar{q}(t)$ as appropriate) is set to the first of the two supplied values (i.e., the second value is ignored), (iii) if $nR > 1$, then the following nR rows define a function that transforms the values read from Column n of **CPCDF.DAT** (i.e., the first value in each row is the value to be transformed and the second value in each row is the transformed value with linear interpolation used to create a continuous transformation function; see example in Fig. 4).

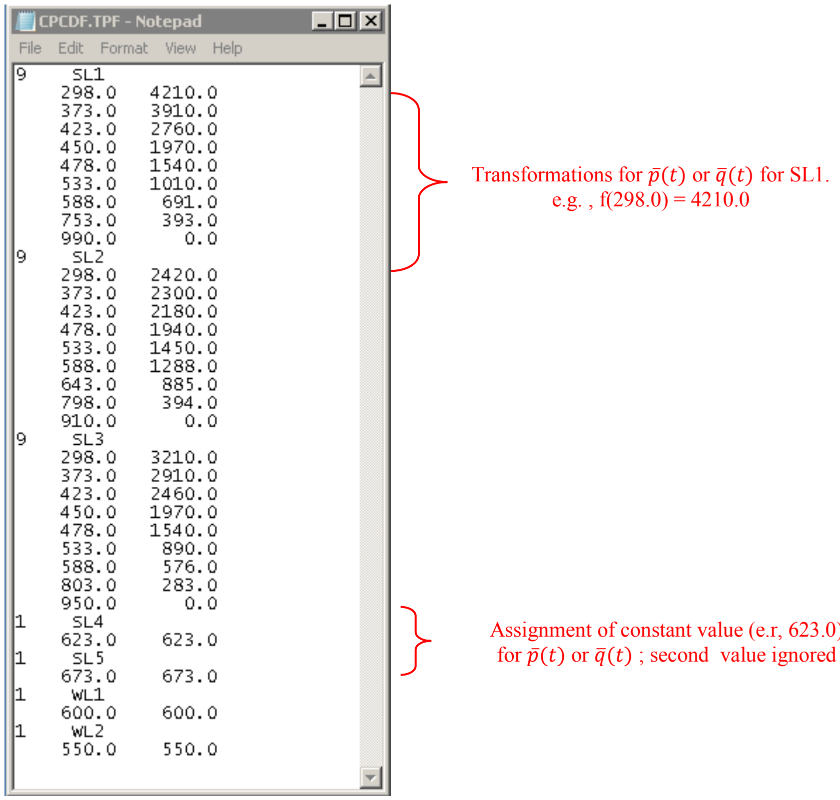


Fig. 3 Example of input file **CPCDF.TPF** that defines transformations of the nominal value functions $\bar{p}(t)$ and $\bar{q}(t)$; see description for Fig. 1, Column g, for additional information.

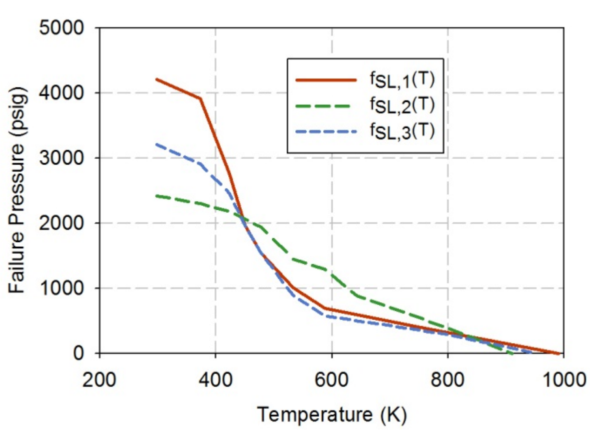


Fig. 4 Example of transformation functions defined in input file **CPCDF.TPF** in Fig. 3 for SL1, SL2 and SL3 for an analysis in which the failure pressure of each link is a function temperature. (i.e., the abscissa corresponds to link temperature and the ordinate corresponds to link failure pressure; see Ref. [8], Sect. 8, for additional discussion of this example).

Fig. 1, Column h: Same as for Column c but for density function $d_{\beta}(\beta)$ indicated in Eq. (2.4) that defines the aleatory uncertainty present in the function $q(t \mid \beta) = \beta\bar{q}(t)$ in Eq. (2.6).

Fig. 1, Columns i-k: Same as for Columns d-f but for the defining parameters for the distributions indicated in Column h. Columns i, j and k contain values for distribution parameters designated by b_1, b_2 and b_3, respectively, for each distribution.

2.2 Definition of WL/SL configurations

For a given set of WLs and SLs, large number of possible WL and SL combinations can be defined. For purposes of terminology, a grouping of 2 or more links with at least one WL and one SL is designated a circuit. The core circuits that CPLOAS_2 can evaluate are listed as Cases 1-4 in Table 1.

The circuits to be considered in a CPLOAS_2 calculation are defined through the input file **CPLOAS_circuit.txt** illustrated in Fig. 5. The structure of **CPLOAS_circuit.txt** is similar to the structure of **CPLOAS_link.txt** and starts with a title and a separator line. Then, each row defines the properties of a single circuit. The content of the individual columns in Fig. 5 is described below.

Table 1 Representation of time-dependent values $pF_i(t)$, i = 1, 2, 3, 4, for PLOAS for WL/SL systems with nWL WLs and nSL SLs and associated verification tests for alternate definitions of LOAS ([9], Table 10; also, Ref. [8],Table 1)

Case 1: Failure of all SLs before failure of any WL (Eqs. (2.1) and (2.5), Ref. [10])

$$pF_1(t) = \sum_{k=1}^{nSL} \left(\int_0^t \left\{ \prod_{\substack{l=1 \\ l \neq k}}^{nSL} CDF_{SL,l}(\tau) \right\} \left\{ \prod_{j=1}^{nWL} \left[1 - CDF_{WL,j}(\tau) \right] \right\} \mathrm{d}CDF_{SL,k}(\tau) \right)$$

Verification test: $pF_1(\infty) = nSL!\,nWL!/(nSL + nWL)!$

Case 2: Failure of any SL before failure of any WL (Eqs. (3.1) and (3.4), Ref. [10])

$$pF_2(t) = \sum_{k=1}^{nSL} \left(\int_0^t \left\{ \prod_{\substack{l=1 \\ l \neq k}}^{nSL} \left[1 - CDF_{SL,l}(\tau) \right] \right\} \left\{ \prod_{j=1}^{nWL} \left[1 - CDF_{WL,j}(\tau) \right] \right\} \mathrm{d}CDF_{SL,k}(\tau) \right)$$

Verification test: $pF_2(\infty) = nSL/(nWL + nSL)$

Case 3: Failure of all SLs before failure of all WLs (Eqs. (4.1) and (4.4), Ref. [10])

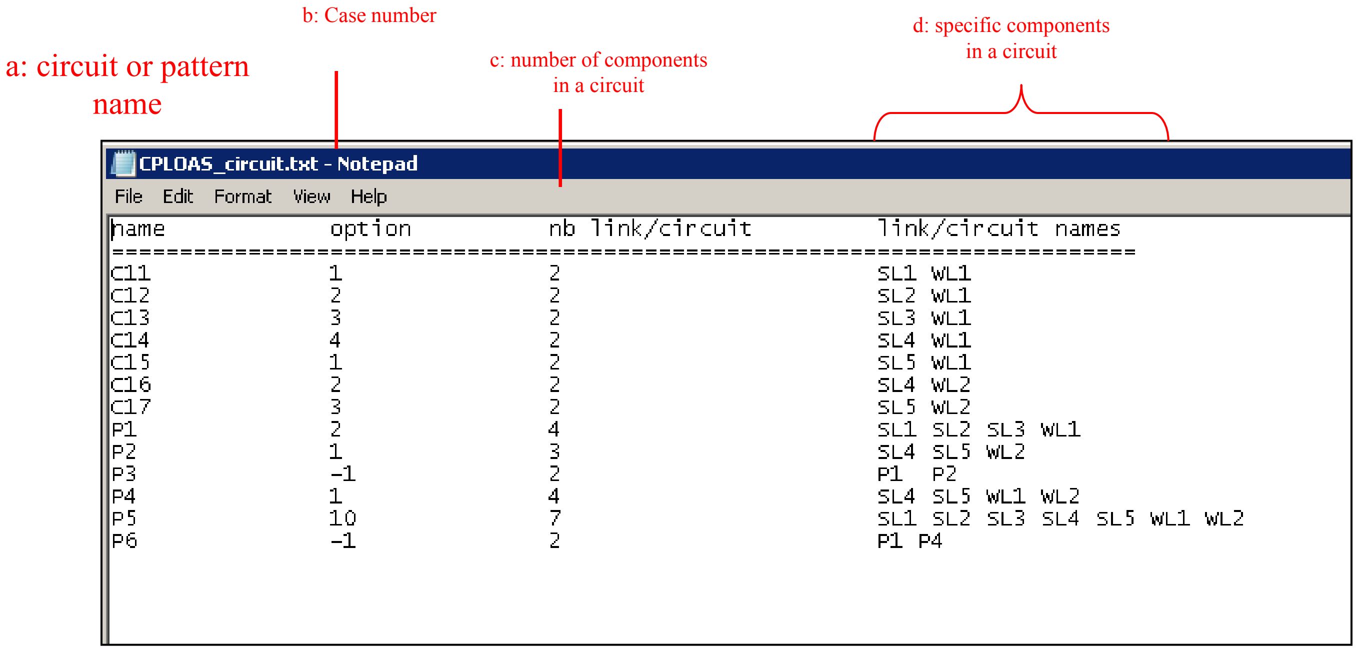
$$pF_3(t) = \sum_{k=1}^{nSL} \left(\int_0^t \left\{ \prod_{\substack{l=1 \\ l \neq k}}^{nSL} CDF_{SL,l}(\tau) \right\} \left\{ 1 - \prod_{j=1}^{nWL} CDF_{WL,j}(\tau) \right\} \mathrm{d}CDF_{SL,k}(\tau) \right)$$

Verification test: $pF_3(\infty) = nWL/(nWL + nSL)$

Case 4: Failure of any SL before failure of all WLs (Eqs. (5.1) and (5.4), Ref. [10])

$$pF_4(t) = \sum_{k=1}^{nSL} \left(\int_0^t \left\{ \prod_{\substack{l=1 \\ l \neq k}}^{nSL} \left[1 - CDF_{SL,l}(\tau) \right] \right\} \left\{ 1 - \prod_{j=1}^{nWL} CDF_{WL,j}(\tau) \right\} \mathrm{d}CDF_{SL,k}(\tau) \right)$$

Verification test: $pF_4(\infty) = 1 - \left[nWL!\,nSL!/(nWL + nSL)! \right]$




```
name          option       nb link/circuit        link/circuit names
============================================================================
C11           1            2                      SL1 WL1
C12           2            2                      SL2 WL1
C13           3            2                      SL3 WL1
C14           4            2                      SL4 WL1
C15           1            2                      SL5 WL1
C16           2            2                      SL4 WL2
C17           3            2                      SL5 WL2
P1            2            4                      SL1 SL2 SL3 WL1
P2            1            3                      SL4 SL5 WL2
P3            -1           2                      P1  P2
P4            1            4                      SL4 SL5 WL1 WL2
P5            10           7                      SL1 SL2 SL3 SL4 SL5 WL1 WL2
P6            -1           2                      P1 P4
```

Fig. 5 Example of input file **CPLOAS_circuit.txt** that defines the circuits to be considered in a CPLOAS_2 calculation.

Fig. 5, Column a: Specifies circuit name for each circuit under consideration. Convention is to use a starting letter “C” for a circuit involving one WL and one SL and starting letter “P” for a circuit involving three or more links (with at least one WL and one SL) but this usage is not mandatory.

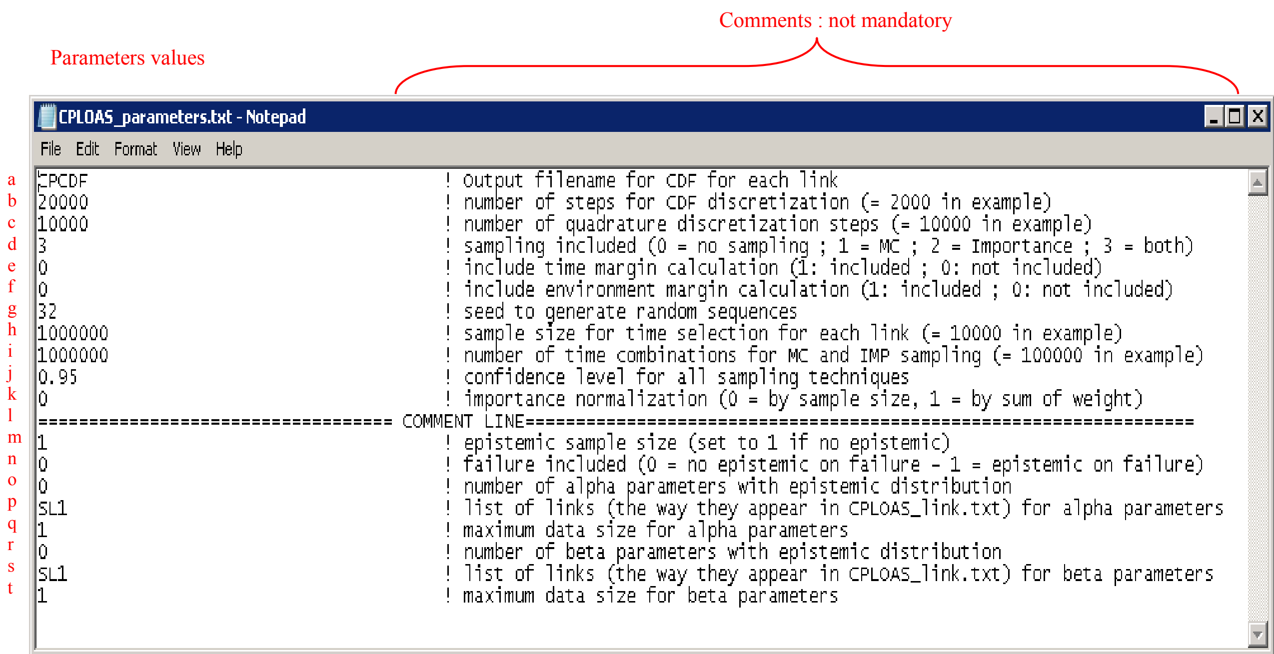
Fig. 5, Column b: Specifies failure pattern of WLs and SLs that defines loss of assured safety (LOAS). Integers 1, 2, 3 and 4 designate failure patterns defined by Cases 1, 2, 3 and 4, respectively, in Table 1 ; 0 designates that the circuits indicated in Column d are assumed to be independent and that LOAS corresponds to all circuits in Column d experiencing LOAS before any one of these circuits is deactivated by failure of WLs; −1 designates that the circuits indicated in Column d are assumed to be independent and that LOAS corresponds to any circuit in Column d experiencing LOAS; and 10 designates a special system defined in Table 6 of Ref. [8] involving 2 WLs and 7 SLs in which LOAS occurs if (i) SL 1, SL 2 or SL 3 fails before WL 1 fails or (ii) SL 4 and SL 5 both fail before WL 1 or WL 2 fails.

Fig. 5, Column c: Specifies number of components in each circuit. For failure patterns designated by integers 1, 2, 3, 4 and 10, this is the number of WLs and SLs in the circuit. For failure patterns designated by 0 and −1, this is the number of subcircuits in the circuit.

Fig. 5, Column d: Specifies Components in a circuit with one space between component names. For failure patterns designated by integers 1, 2, 3, 4 and 10, this is a listing of the WLs and SLs in the circuit. For failure patterns designated by 0 and −1, this is a listing of the subcircuits in the circuit. Order of components in list not important.

2.3 Control of Calculations

The performance of calculations by CPLOAS_2 is controlled by analysis properties specified in the input file **CPLOAS_parameters.txt** illustrated in Fig. 6. Properties must be listed in order shown in Fig. 6. The content of the individual rows in Fig. 6 is described below.




```
CPCDF                                    ! output filename for CDF for each link
20000                                    ! number of steps for CDF discretization (= 2000 in example)
10000                                    ! number of quadrature discretization steps (= 10000 in example)
3                                        ! sampling included (0 = no sampling ; 1 = MC ; 2 = Importance ; 3 = both)
0                                        ! include time margin calculation (1: included ; 0: not included)
0                                        ! include environment margin calculation (1: included ; 0: not included)
32                                       ! seed to generate random sequences
1000000                                  ! sample size for time selection for each link (= 10000 in example)
1000000                                  ! number of time combinations for MC and IMP sampling (= 100000 in example)
0.95                                     ! confidence level for all sampling techniques
0                                        ! importance normalization (0 = by sample size, 1 = by sum of weight)
=================================== COMMENT LINE================================================================
1                                        ! epistemic sample size (set to 1 if no epistemic)
0                                        ! failure included (0 = no epistemic on failure - 1 = epistemic on failure)
0                                        ! number of alpha parameters with epistemic distribution
SL1                                      ! list of links (the way they appear in CPLOAS_link.txt) for alpha parameters
1                                        ! maximum data size for alpha parameters
0                                        ! number of beta parameters with epistemic distribution
SL1                                      ! list of links (the way they appear in CPLOAS_link.txt) for beta parameters
1                                        ! maximum data size for beta parameters
```

Fig. 6 Example of input file **CPLOAS_parameters.txt** that defines analysis properties that control the performance of calculations by CPLOAS_2.

Fig. 6, **Row a:** Name (i.e., ***name***) of thermo-pressure history file (without extension ***k*** as described below). The code will look for the file ***name***.DAT of the form shown in Fig. 2 in the absence of a propagation of epistemic uncertainty and for a set of files ***namek***.DAT where ***k*** corresponds to a sample element used in the propagation of epistemic uncertainty (see row h description below). This string will also be used as a prefix for the generated output files (see Sect. 3).

Fig. 6, **Row b:** Number *nCDF* of steps for CDF discretization. Integer > 0; set to *nCDF* = 20000 in example. Controls resolution of the discretization used to characterize the CDF for probability of failure as a function of time for each link for use in determination of PLOAS (i.e., each CDF is approximated by probability of failure values for *nCDF* equally spaced time steps). See Ref. [8], Sect. 2, for additional information on indicated CDFs.

Fig. 6, **Row c:** Number *nQUAD* of quadrature discretization steps. Integer > 0; set to *nQUAD* = 10000 in example. This is the discretization used at each time step for the quadrature procedure used to estimate PLOAS for each circuit (i.e., PLOAS at time t is approximated with *nQUAD* equally spaced time intervals over the time interval $[t_{mn}, t]$). See Ref. [8], Sect. 4, for additional information on quadrature procedures.

Fig. 6**, Row d:** Flag indicating if sampling-based procedures in addition to quadrature-based procedures are to be used in the determination of PLOAS, with 0 ~ only quadrature-based procedures, 1 ~ random sampling procedures 1 and 2 included (see Sect. 4 for additional discussion of random sampling procedures), 2 ~ importance sampling procedures 1 and 2 included (see Sect. 4 for additional discussion of importance sampling procedures), and 3 ~ all four sampling procedures included. This option also specifies the numerical procedures used to determine PLOAS for analyses involving epistemic uncertainty (i.e., with *nEUS* > 1 – see row k below)

Fig. 6**, Row e:** Flag indicating if time-margin calculations for each circuit will be performed, with 0 ~ no time margin calculation and 1~ time margin calculation.

Fig. 6**, Row f:** Flag indicating if environmental -margin calculations for each circuit will be performed, with 0 ~ no environmental margin calculation and 1~ environmental margin calculation.

Fig. 6**, Row g:** Random seed *nRSEED* used to initiate generation of random sequences for use in Monte Carlo and importance sampling procedures. Integer; set to *nRSEED* = 32 in example. A unique random seed is used to initiate generation of random sequences for simple random sampling and importance sampling. See Ref. [8], Sect. 5, for additional information on simple random sampling and importance sampling procedures.

Fig. 6**, Row h:** Sample size *nFT* for generating failure times for each link for simple random sampling and importance sampling. Integer > 0; set to *nFT* = 1,000,000 in example. Specifically, *nFT* failure times are initially sampled for each link. Next, *nFTC* failure time combinations over all links are sampled from the *nFT* failure times for the individual links. Then, the *nFTC* failure time combinations over all links are used in the determination of PLOAS. See Ref. [8], Sect. 5, for additional information on simple random sampling and importance sampling procedures.

Fig. 6**, Row i:** Sample size *nFTC* for generating failure time combinations over all links for use in the determination of PLOAS with simple random sampling and importance sampling procedures. Integer > 0; set to *nFTC* = 1,000,000 in example. See description of Fig. 6, row h, for additional information. This is also the sample size used to generate the time margin (resp. environmental margin) results if option is set to 1 in Fig. 6 row e (resp. Fig. 6 row f)

Fig. 6**, Row j:** Confidence level for all Monte Carlo and Importance sampling techniques. When these methods are selected, the confidence level is used to estimate a confidence interval for the expected value. For a confidence level $0 < q < 1$ the confidence interval $\left[\frac{1-q}{2}, \frac{1+q}{2}\right]$ will be estimated (for instance, a confidence level of 0.95 will estimate the confidence interval [0.025,0.975]). For $q \leq 0$ or $q \geq 1$ the estimate of mean and standard deviation are reported.

Fig. 6**, Row k:** Flag indicating the normalization method used for the importance sampling. With 0~ the value is normalized by the sample size and 1~ the value is normalized by the sum of weights. The recommendation is to leave this value at 0.

Fig. 6**, Row l:** Comment line (that must be present) that separates the first set of parameters from a second set. The second parameter set defines options related to the incorporation of epistemic uncertainty into PLOAS results. The indicated information must be entered in the order shown below. Multiple examples of PLOAS analyses involving epistemic uncertainty are given in Sects. 10 and 11 of Ref. [8].

Fig. 6**, Row m:** Epistemic uncertainty sample size *nEUS*. Integer between 1 and 9999 with *nEUS* = 1 indicating that there is no consideration of epistemic uncertainty. For *nEUS* > 1, input files ***namek*.DAT**, k = 1, 2, …, *nEUS*, must be supplied that (i) define time dependent system properties for each epistemic uncertainty sample element and (ii) have the same structure as the input file ***name*.DAT** illustrated in Fig. 2. If *nEUS* = 1, other variables (except for row j) are read and not used.

Fig. 6**, Row n:** Flag *nFV* indicating if epistemic uncertainty is present in one or more trasformations of the nominal link properties $\overline{p}(t)$ and $\overline{q}(t)$ indicated in Eqs. (2.1) and (2.2) , with the absence and presence of epistemic uncertainty in link failure values indicated by *nFV* = 0 and *nFV* = 1, respectively. For *nFV* = 1, input files ***namek*.TPF**, k = 1, 2, …, *nEUS*, must be supplied that (i) define link failure values for each epistemic uncertainty sample element and (ii) have the same structure as the input file ***name*.TPF** illustrated in Fig. 3. Note that transformation (or constant) defined in the TPF file can be used for each link to either its property or its failure, but not on both.

Fig. 6**, Row o:** Number *nEAD* of distributions characterizing aleatory uncertainty (i.e., α values; see Eq. (2.3) and Ref. [8], Sect. 2) in link physical properties that have epistemic uncertainty present in their definitions, with *nEAD* = 0 indicating that there is no epistemic uncertainty present in the definitions of the distributions characterizing aleatory uncertainty in link physical properties.

Fig. 6**, Row p:** List of the *nEAD* links with epistemic uncertainty present in the definitions of the distributions characterizing aleatory uncertainty in their physical properties, with individual links represented by the same character strings specified in the input file **CPLOAS_link.txt** that defines WL and SL properties (e.g., SL1 and WL1 in this example). A single link name should be entered as a place holder if *nEAD* = 0. For *nEAD* > 0, an input file ***name*_LINKNAME_alpha.dat** with the structure of the file shown in Fig. 7 must be defined for each of the *nEAD* distributions with epistemic uncertainty in their definitions. For each of the *nEAD* links, the actual link name replaces "**LINKNAME**" in the corresponding input file. Thus, for this example, the file names corresponding to the specified links SL1 and WL1 would be **SL1_alpha.dat** and **WL1_alpha.dat**, respectively.

The file ***name*_LINKNAME_alpha.dat** is structured as follows: (i) First row lists the number of observations and (ii) each following row is a single observation. In order, each row contains the following: (i) An integer designator for distribution type (see description for Fig. 1, Column c), (ii) Defining parameters a_1, a_2 and a_3 for specified distribution type (see description for Fig. 1, Columns d-f), and (iii) a weight associated with the observation in the row. In most analyses, the number of observations specified in this file will be the same as the epistemic uncertainty sample size *nEUS* indicated in Fig. 6, row m, and the indicated weight will be 1/*nEUS*. If the sample size is equal to *nEUS*, values will be used in order for each epistemic sample element. If it differs, the code will randomly sample from the set of values, according to the reported weight.

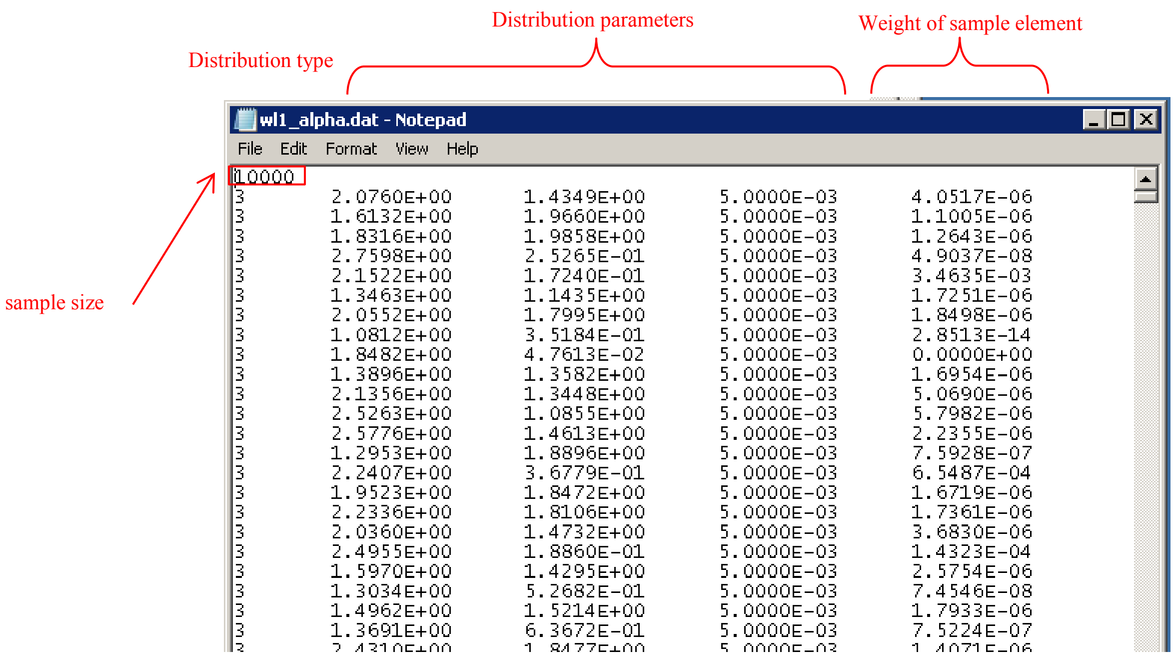


Fig. 7 Example of input file ***name*_LINKNAME_alpha.dat** containing sampled values for uncertain quantities present in the definition of a distribution characterizing aleatory uncertainty in the physical properties of a WL or a SL; for a specific link, the name of that link replaces "**LINKNAME**" in the file name.

Fig. 6, Row q: Maximum data size *nMDSA* for alpha parameters data set (integer *nMDSA* > 0). Corresponds to maximum number of observations in the files ***name*_LINKNAME_alpha.dat** (see description for Fig. 6, row l). This parameter is requested to simplified the creation of multidimensional arrays without a requirement to read all file lengths.

Fig. 6, Row r: Number *nEBD* of distributions characterizing aleatory uncertainty (i.e., β values; see Eq. (2.4) and Ref. [8], Sect. 2) in link failure values that have epistemic uncertainty present in their definitions, with *nEBD* = 0 indicating that there is no epistemic uncertainty present in the definitions of the distributions characterizing aleatory uncertainty in link failure values.

Fig. 6, Row s: List of the *nEBD* links with epistemic uncertainty present in the definitions of the distributions characterizing aleatory uncertainty in their failure values, with individual links represented by the same character strings specified in the input file **CPLOAS_link.txt** that defines WL and SL properties. A single link name should be entered as a place holder if *nEBD* = 0. For *nEBD* > 0, an input file ***name*_LINKNAME_beta.dat** with the structure of the file shown in Fig. 7 must be defined for each of the *nEBD* distributions with epistemic uncertainty in their definitions. Additional discussion same as for ***name*_LINKNAME_alpha.dat** in description for Fig. 6, row l.

Fig. 6**, Row t:** Maximum data size *nMDSB* for beta parameters data set (integer *nMDSB* > 0). Corresponds to maximum number of observations in the files ***name*_LINKNAME_beta.dat** (see description for Fig. 6, row l). This parameter is requested to simplified the creation of multidimensional arrays without requiring to read all file lengths.

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3 OUTPUT FILES GENERATED BY CPLOAS_2

As described in Sects. 3.1 and 3.2, results are saved and displayed differently depending whether or not an analysis of epistemic uncertainty is performed.

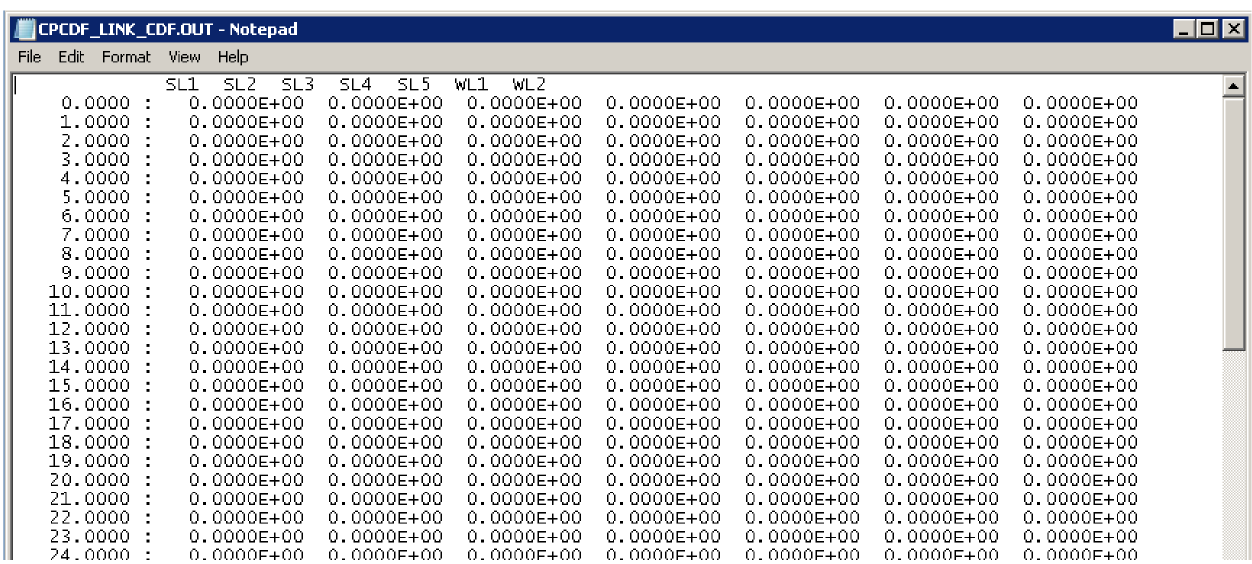
3.1 Epistemic uncertainty not considered

In this configuration (i.e., epistemic sample size set to *nEUS* = 1 as described for Fig. 6, Row h), six files are saved, with names based on the name specified in the first row of **CPLOAS_parameters.txt**.

First file (default name: *name*_**LINK_CDF.OUT**, where *name* is specified in Row a of Fig. 6) lists the time-dependent probability of failure for each link defined in **CPLOAS_link.txt**. The first row lists the link names. Subsequent rows list the time-step (first column) and cumulative probability of failure for each link at this time-step. Cumulative failure properties are listed in the same order as link names in the first row; in turn, the order of the link names is the same as the order in which they are specified in Column a of Fig. 1. An example of the file *name*_**LINK_CDF** is given in Fig. 8.

Second file (default name: *name*_**PLOAS_QUAD.OUT**) lists estimated PLOAS values obtained using quadrature for each circuit specified in **CPLOAS_circuit.txt**. The first row lists the circuit names. Subsequent rows list the time (first column) and cumulative probability of failure for each circuit at this time. An example is shown in Fig. 9.

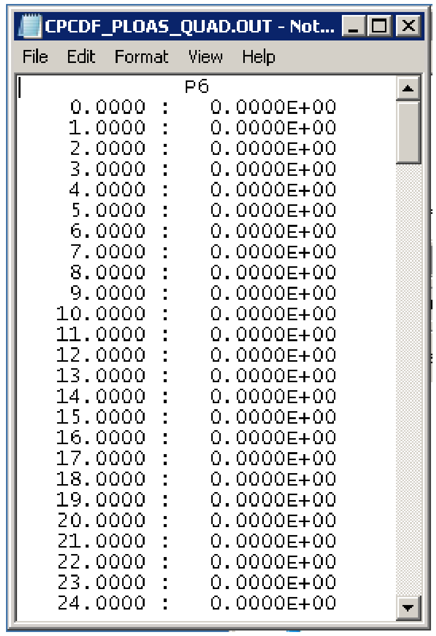
Timestep CDFs for all links defined in CPLOAS_link.txt (see Fig. 1)


CPCDF_LINK_CDF.OUT - Notepad

File Edit Format View Help

		SL1	SL2	SL3	SL4	SL5	WL1	WL2
0.0000	:	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
1.0000	:	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
2.0000	:	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
3.0000	:	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
4.0000	:	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
5.0000	:	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
6.0000	:	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
7.0000	:	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
8.0000	:	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
9.0000	:	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
10.0000	:	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
11.0000	:	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
12.0000	:	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
13.0000	:	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
14.0000	:	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
15.0000	:	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
16.0000	:	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
17.0000	:	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
18.0000	:	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
19.0000	:	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
20.0000	:	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
21.0000	:	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
22.0000	:	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
23.0000	:	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
24.0000	:	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00

Fig. 8 Example of output file *name*_**LINK_CDF.OUT** containing time-dependent link failures at each time-step for all considered links. The name of the analysis (Fig. 6 row a) replaces *name* in the file name.




```
                 P6
   0.0000 :    0.0000E+00
   1.0000 :    0.0000E+00
   2.0000 :    0.0000E+00
   3.0000 :    0.0000E+00
   4.0000 :    0.0000E+00
   5.0000 :    0.0000E+00
   6.0000 :    0.0000E+00
   7.0000 :    0.0000E+00
   8.0000 :    0.0000E+00
   9.0000 :    0.0000E+00
  10.0000 :    0.0000E+00
  11.0000 :    0.0000E+00
  12.0000 :    0.0000E+00
  13.0000 :    0.0000E+00
  14.0000 :    0.0000E+00
  15.0000 :    0.0000E+00
  16.0000 :    0.0000E+00
  17.0000 :    0.0000E+00
  18.0000 :    0.0000E+00
  19.0000 :    0.0000E+00
  20.0000 :    0.0000E+00
  21.0000 :    0.0000E+00
  22.0000 :    0.0000E+00
  23.0000 :    0.0000E+00
  24.0000 :    0.0000E+00
```

Fig. 9 Example of output file *name*_**PLOAS_QUAD.OUT** containing PLOAS estimated at each time-step for all considered circuits. The name of the analysis (Fig. 6, Row a) replaces *name* in the file name.

Third, Fourth, Fifth and Sixth output files (respective default names: *name*_**PLOAS_MC.OUT** *name*_**PLOAS_MC_2.OUT,** *name*_**PLOAS_IMP.OUT** and *name*_**PLOAS_IMP_2.OUT**) list results obtained with, respectively, random sampling procedures 1 and 2 and importance sampling procedures 1 and 2 (see Sect. 4 for a discussion of sampling-based procedures for the determination of PLOAS). The last four files will always be created but filled only if the option (from Fig. 6, Row j) is set so that PLOAS values are actually calculated using these techniques. Since CPLOAS version 2.10 the file format is different to the one presented in Fig. 9 as either confidence intervals around the mean are displayed (confidence level in]0,1[ ) or the mean and standard deviation (confidence level outside of this range). An example is shown in Fig. 10.

```
CPCDF_PLOAS_MC.OUT - Notepad
File  Edit  Format  View  Help
                          P1                          P2                          P3                          P4
  0.0000 :    0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
  0.1000 :    0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
  0.2000 :    0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
  0.3000 :    0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
  0.4000 :    0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
  0.5000 :    0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
  0.6000 :    0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
  0.7000 :    0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
  0.8000 :    0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
  0.9000 :    0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
  1.0000 :    0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
  1.1000 :    0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
  1.2000 :    0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
  1.3000 :    0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
  1.4000 :    0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
  1.5000 :    0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
  1.6000 :    0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
  1.7000 :    0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
  1.8000 :    0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
  1.9000 :    0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
  2.0000 :    0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
  2.1000 :    0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
  2.2000 :    0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
  2.3000 :    0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
  2.4000 :    0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
  2.5000 :    0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
```

Fig. 10: Example of output file *name*_**PLOAS_MC.OUT** containing PLOAS estimated using Monte Carlo approach at each time-step for all considered circuits. The name of the analysis (Fig. 6, Row a) replaces *name* in the file name.

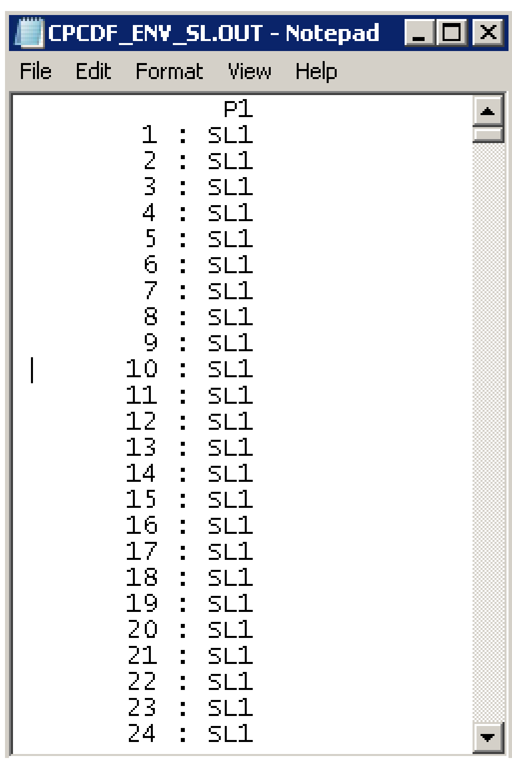
```
CPCDF_TIME.OUT - Notepad
File  Edit  Format  View  Help
                   P1
          1 :    0.6384E+01
          2 :    0.4426E+01
          3 :    0.6072E+01
          4 :    0.7813E+01
          5 :    0.5432E+01
          6 :    0.3880E+01
          7 :    0.6551E+01
          8 :    0.4617E+01
          9 :    0.3024E+01
         10 :    0.5634E+01
         11 :    0.7279E+01
         12 :    0.8251E+01
         13 :    0.3652E+01
         14 :    0.6333E+01
         15 :    0.5511E+01
         16 :    0.3282E+01
         17 :    0.5571E+01
         18 :    0.6128E+01
         19 :    0.3968E+01
         20 :    0.4503E+01
         21 :    0.4381E+01
         22 :    0.7649E+01
         23 :    0.2916E+01
         24 :    0.5466E+01
         25 :    0.3950E+01
         26 :    0.2836E+01
```

Fig. 11. Example of output file *name*_**TIME.OUT** containing Time margin estimated for different realization for all considered circuits. The name of the analysis (Fig. 6, Row a) replaces *name* in the file name.

Since version 2.7 CPLOAS also estimates time and environmental margins when required by the user. Time margin results are saved in *name*_**TIME.OUT** . The time margin is not a time-dependent result and the file reflects the estimate of time margin for *nFTC* (see Fig. 6 row i) random realizations. An example is shown in Fig. 11.

Environmental margin results are saved in *name*_**ENVIR.OUT** . The environmental margin is not a time-dependent result and the file reflects the estimate of time margin for *nFTC* (see Fig. 6 row i) random realizations. The file format is similar to the one presented in Fig. 11.

When environmental margin is calculated for a circuit, the code still look at the most appropriate distance between strong link and weak link (based on the circuit type considered), so it is possible for environmental margin to be reported for different strong links. As different properties can be used for different strong link (for instance temperature or pressure), the user may want to sort the environmental margin based on the strong link that was used to generate useful results. Consequently, another file is created for environmental margin, whose name is in *name*_**ENV_SL.OUT** . This file lists, for each realization, which strong link was used. An example of such file is shown in Fig. 12. Note that the only information used with respect to weak link to estimate environmental margin is the time of weak link failure. Thus, there is no need to track which weak link was involved in the estimate environmental margin.




```
           P1
   1  :  SL1
   2  :  SL1
   3  :  SL1
   4  :  SL1
   5  :  SL1
   6  :  SL1
   7  :  SL1
   8  :  SL1
   9  :  SL1
  10  :  SL1
  11  :  SL1
  12  :  SL1
  13  :  SL1
  14  :  SL1
  15  :  SL1
  16  :  SL1
  17  :  SL1
  18  :  SL1
  19  :  SL1
  20  :  SL1
  21  :  SL1
  22  :  SL1
  23  :  SL1
  24  :  SL1
```

Fig. 12. Example of output file *name*_**ENV_SL.OUT** containing which strong link was used to estimate environmental margin for each realization and for all considered circuits. The name of the analysis (Fig. 6, Row a) replaces *name* in the file name

3.2 Epistemic uncertainty considered

If epistemic sample size is set to a number greater than 1 (i.e., epistemic sample size set to *nEUS* > 1 as described for Fig. 6, Row h), then results are saved differently. Use of the same file structure as described in Sect. 3.1 would lead to one set of files for each epistemic sample element. It is common to consider epistemic samples of size 100 or more, which would generate hundreds or thousands of files and would not be practical.

Instead, it is more appropriate to group results by links and circuits. With this approach, a file is created for each link (saving results that were saved in the first file of described in Sect. 3.1; i.e., the file *name*_**LINK_CDF.OUT**). The resulting file for each link has a name that begins with the analysis name *name* specified in **CPLOAS_link.txt** and followed by the link name and the suffix ".out" (e.g., **CPCDF_SL3.out**). This file will have as many columns as epistemic sample elements plus 1. The first column corresponds to the time steps. Subsequent columns list cumulative failure probability for this link for the corresponding epistemic element as shown in Fig. 13.

For circuits, the same approach is used, except that the circuit name is used instead of a link name and the suffix used to create the output file name is "_quad.out" (e.g., *name*_**P1_QUAD.OUT** for circuit P1) for quadrature results (always the case). Similarly, the suffices "_MC.out" and "MC_2.out" are used for results obtained with random sampling procedures 1 and 2 (if the option is set to generate these results), and the suffices "_imp.out" and "_imp_2.out" are used for results obtained with importance sampling procedures 1 and 2 (if the option is set to generate these results); see Sect. 4 for a discussion of sampling-based procedures for the determination of PLOAS. Results are saved as illustrated in Fig. 13.

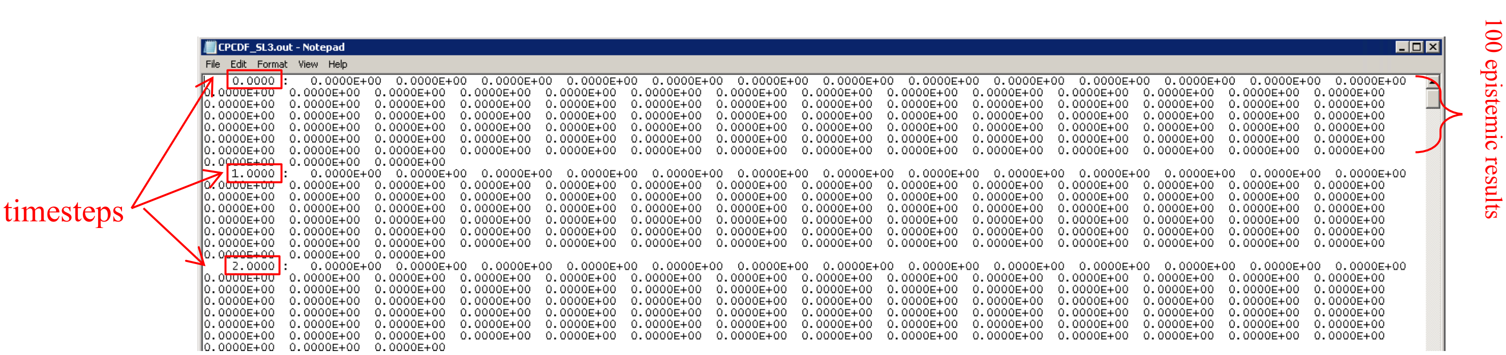


Fig. 13 Example of link failure result file (for SL3 in this example) showing the first 3 timesteps and using a sample of size 100 (Note: columns wrap with the result that 101 columns appear as 8 rows).

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4 SOME CONSIDERATIONS IN THE USE OF SAMPLING IN CPLOAS_2

4.1 Simple Random Sampling from CDFs for Link Failure Times

As described in conjunction with Eq. (5.9) of Ref. [8], simple random sampling from the CDFs for link failure time can be used in the estimation of PLOAS. Further, as described in conjunction with Eqs. (5.20)-(5.28) Ref. [8], this sampling can be performed as a two stage process in which *nFT* link failure times are randomly selected for each link from the corresponding link failure time CDFs and then *nFTC* combinations of link failure times are randomly selected from the previously sampled *nFT* failure times for each link. Specifically, if *nWL* WLs and *nSL* SLs are under consideration, the second stage of the sampling results in *nFTC* vectors of length $nL = nWL + nSL$ that contain one failure time for each link. In turn, these vectors of link failure times can be used as indicated in Eq. (5.9) of Ref. [8] to determine PLOAS as a function of time.

The preceding two stage procedure is used in CPLOAS_2 for the estimation of PLOAS with simple random sampling from the failure time CDFs for the individual links. This procedure is referred to as random sampling procedure 1; see Sect. 4.3 for a discussion of random sampling procedure 2.

The values used for *nFT* and *nFTC* are set in rows e and f of Fig. 6. Further, the indicated CDFs are estimated with quadrature procedures and saved at *nCDF* evenly spaced times, with the value for *nCDF* set in row b of Fig. 6. The sample sizes *nFT* and *nFTC* selected for use in a given analysis depend on both the desired accuracy in the determination of PLOAS and the probability of failure for the individual links. Sample of sizes of $nFT = 100$ and $nFTC = 1000$ will be enough for links that have large probabilities of failing during a simulation time (e.g., 0.2), but will not be appropriate for links that have 0.01 probability of failure over time.

As an example, the three links with the properties and failure values shown in Fig. 14 are used for illustration. The probability of failure for a given link corresponds to the likelihood of being in a situation where a red curve (representing a time-dependent failure value) crosses a green curve (representing a time dependent system property). For SL2 (frame b), this situation will never occur in the time period under consideration and any sample size will be appropriate. The probability of failing for SL3 (frame c) starts around 150 minutes and increase gradually to 0.1 at the end of simulation (200 minutes). A sample of size $nFT = 10{,}000$ will generate about 1000 failure times and 9000 non failure times with simple random sampling. For SL1 (frame a), failure occurs late (after 190 minutes) with a probability of about 0.002 at the end of the simulation. In this case, a sample of size $nFT = 10{,}000$ will only generate about 20 failure times with simple random sampling.

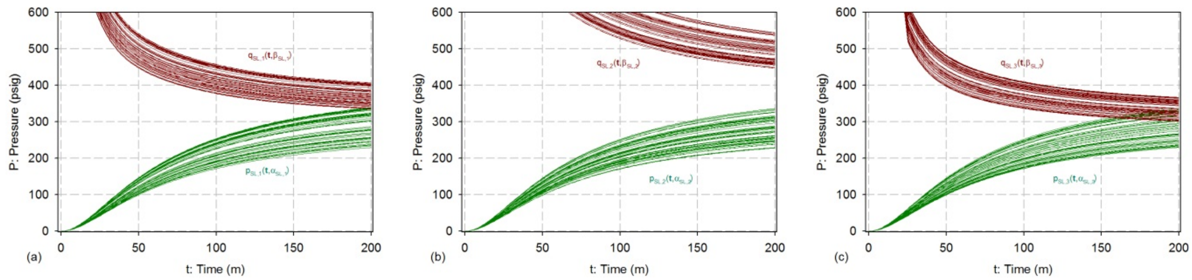

Fig. 14 Time-dependent property and failure values for three links: (a) SL1, (b) SL2 and (c) SL3.

The sample size will affect how many failures are generated but also, for unlikely failures, when the first failure will occur. As an example, the time-dependent probability of failure (estimated using quadrature) for SL1 is reported in Table 2. The probability of having a failure before 192 min is a little less than 10^{-5}. As a result, using a sample of size 10,000 to generate an array of failure times (via Monte Carlo) is unlikely to generate a failure time prior to 192 min and, at best, one or two such failure times will be present in a sample of size 10,000. As a consequence, the sample size (i.e., *nFT*) may need to be greater than 10,000 depending on how much accuracy is required in the determination of PLOAS for a circuit involving SL1.

Table 2 Time dependent probability of failure for SL1 using quadrature

time	prob. Failure SL1
191	0.00E+00
192	9.36E-06
193	7.49E-05
194	2.01E-04
195	3.86E-04
196	6.28E-04
197	9.24E-04
198	1.27E-03
199	1.67E-03
200	2.12E-03

In conclusion, with respect to choosing appropriate values *nFT* and *nFTC*, a possible strategy is to initially examine the link failure probabilities (in the file **LINK_CDF.OUT** in the example set) to determine if small failure probabilities are present, which would imply that larger rather than smaller values for *nFT* and *nFTC* are needed. Then, increasing values for *nFT* and *nFTC* could be tried until the estimates for PLOAS show little change with increasing values for *nFT* and *nFTC*. Although not a currently defined option in CPLOAS_2, PLOAS could be repeatedly evaluated with the same values for *nFT* and *nFTC* but with different random seeds to start the random sampling process (see *RSEED* in row d of Fig. 6) and then a confidence interval calculated around the mean PLOAS value from these multiple CPLOAS_2 runs.

4.2 Importance Sampling from CDFs for Link Failure Times

Importance sampling from CDFs for link failure times in CPLOAS_2 uses the same two stage sampling procedure described in Sect. 4.1 for simple random sampling from CDFs for link failure. However, as described in the next paragraph, the sampling of link failure times at the first stage of this two stage process is not is where the importance sampling is implemented.

As implemented in CPLOAS_2, importance sampling from link failure time CDFs uses a left-triangular distribution on [0, 1] with mode at 0 to sample failure times for SLs and a right-triangular distribution on [0, 1] with mode at 1 to sample failure times for WLs to generate this number. As a result, early failure times are over sampled for SLs and late failure times are over sampled for WL. A weight is associated with each sampled failure time to correct for the indicated over sampling of early and late failure times for SLs and WLs, respectively (see discussion associated with Eqs. (5.29)- (5.31) of Ref. [8]). In turn, this weight is used when sampling the individual link failure times at the second stage of the two stage sampling process. This procedure is referred to as importance sampling procedure 1.

To test the efficiency of this technique, a circuit with low probability of failure (on the order of 10^{-4} for PLOAS after 200 minutes) has been considered. Specifically, 30 estimates (using different random seeds) of PLOAS have been generated for both importance sampling and simple random using the same properties (i.e., nFT = 10,000 and $nFTC$ = 100,000). The resultant time-dependent mean for PLOAS as well as a confidence intervals for both sampling techniques are shown in Fig. 15 and compared to the quadrature result. Plain green and red lines represent mean values for simple random sampling and importance sampling, respectively. Both lines are close to the quadrature results, although the importance sampling means seems to be more accurate than simple random sampling mean at the end of simulation (i.e., at 200 min). Dashed and dotted lines represent confidence intervals over the results and are indicators of the accuracy of each sampling method. The red lines (for importance sampling) are closer together than the green lines for simple random sampling and thus indicate that the importance sampling results are more precise than the simple random sampling results.

As both simple random (i.e., Monte Carlo) and importance sampling use the same procedure to generate an initial set of failure times (i.e., importance sampling is not used in Step 1 of the two step sampling procedure) and no failure was generated prior to 192 min with a sample of size 10,000, both methods cannot match with quadrature results prior to 192 min. Increasing the initial (i.e., Step 1) sample size to 100,000 would lead to a better estimate at early times in this example. In the future, the CPLOAS_2 may be modified to include importance sampling at this first step to increase resolution in estimates for PLOAS at early times. However, PLOAS values at the end of an accident are usually the result of greatest interest rather than PLOAS values early in the development of an accident.

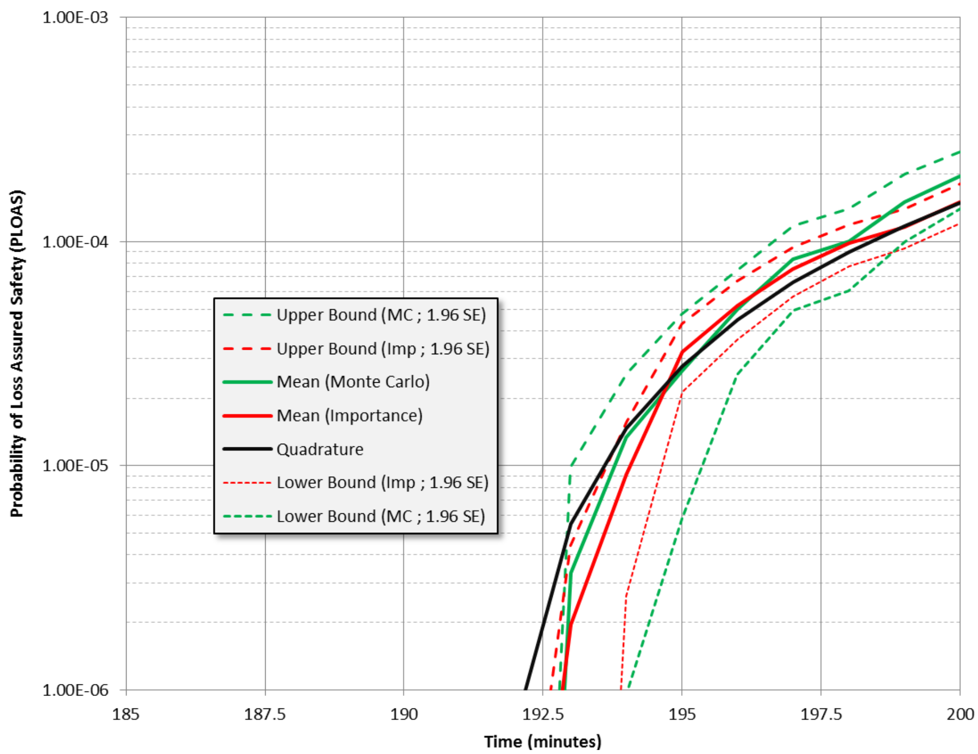

Fig. 15 Mean and q = 0.9 confidence interval for 30 replicates of random sampling procedure 1 and importance sampling procedure 1 for circuit C11 defined in Fig. 5.

4.3 Simple Random and Importance Sampling from Aleatory Variables α and β

An additional option in CPLOAS_2 is to directly sample from the aleatory variables α and β used in the definitions of link properties and failure values as indicated in Eqs. (2.5) and (2.6). This sampling can be done with either simple random sampling or importance sampling as described in conjunction with Eqs. (5.18) and (5.19) of Ref. [8]. The determination of PLOAS with direct sampling of the aleatory variables α and β in CPLOAS_2 does not use the two stage sampling procedure described in Sect. 4.1. Rather, a single sampling of the aleatory variables α and β and determination of associated link failure times is performed; then, PLOAS is determined as indicated in Eqs. (5.18) and (5.19) of Ref. [8].

The sampling procedure described in conjunction with Eq. (5.18) of Ref. [8] is referred to as sampling or Monte Carlo procedure 2, and the sampling procedure described in conjunction with Eq. (5.19) of Ref. [8] is referred to as importance sampling procedure 2. The manner in which the sampling-based procedures are specified for use in CPLOAS_2 is described in the discussion for Fig. 6, row j.

4.4 Choice of sampling method based on PLOAS value of interest

The sampling procedures have been developed in CPLOAS to increase confidence in the quadrature results and check that different (although not as efficient) approaches lead to similar results. Results can be compared directly if mean and standard deviation are displayed for the sampling method: the mean value is equivalent to the PLOAS value estimated via quadrature. If a confidence interval is displayed, one can check whether the PLOAS value estimated with quadrature is included within the confidence interval.

One first important caveat is that it is not guarantee that the PLOAS value in the quadrature will be included within the confidence interval. The first reason is that the quadrature is still a numerical technique with a certain level of accuracy and the error may be enough to have the value slightly biased. The second is that the Monte Carlo and Importance techniques have some assumptions that may also bias the results (for instance, when a Strong Link and a Weak Link fail during the same time-step, there is an assumption that the Strong Link fails first, which may induce a bias for extremely low value of PLOAS)

The second important caveat is that the use of Importance sampling is not always appropriate for comparison. In order to make the use of importance sampling not too complex for the user, some assumptions have been made during the development phase. These assumptions restrict the use of importance sampling to a certain range of PLOAS value. As displayed in Fig. 16, the first importance sampling technique gives more accurate results than classical Monte Carlo for PLOAS values below 10^{-2}. The second importance technique creating more extreme cases starts to give better results for PLOAS values around 10^{-5}

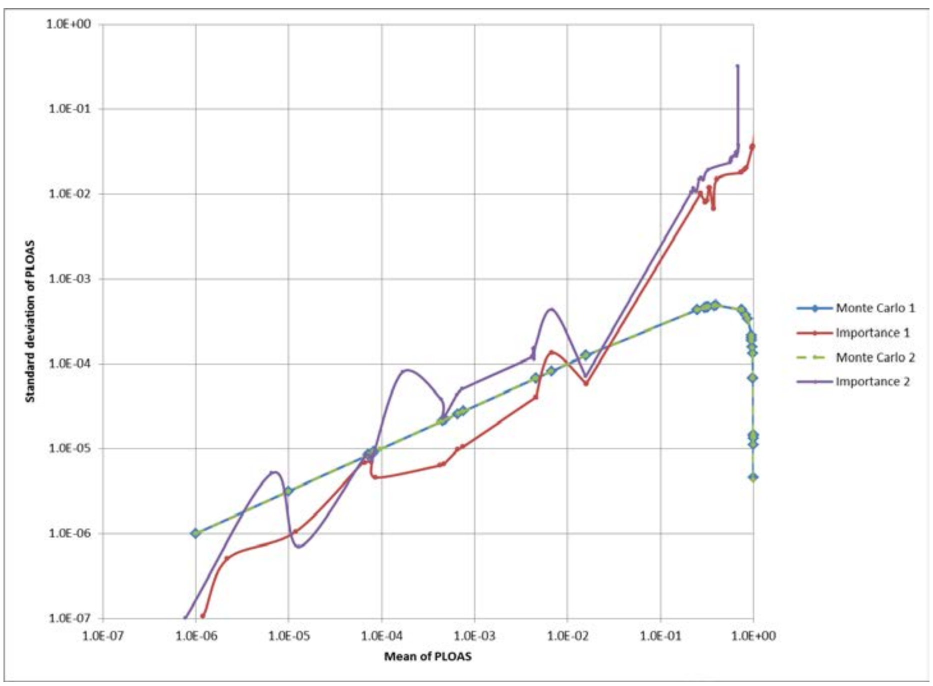

Fig. 16: standard deviation of the results as function of the mean value for the four considered techniques

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5 USE OF CPLOAS2

This section describes the step by step procedure the user has to follow in order to create the input files and run the code with the appropriate parameters.

5.1 Generation of time-dependent properties

The first file required for the analysis lists the time-dependent properties generated or measured for the set of components. This file is presented in Fig. 2 and described in the description of Fig. 1 column b. User can select different name for this file (CPCDF was used as an example) but the extension has to be ".DAT" and a matching name will have to be used in the CPLOAS_Parameters file.

5.2 Definition of Strong and Weak Links and creation of *CPLOAS_link.txt*

The next step is to define the system as a collection of strong links and weak links. Any CPLOAS2 run will require at least one strong link and one weak link.

The naming convention used and recommended was "SL" for Strong link and "WL" for Weak link, each followed with up to 3 digits identifying the link with a number. The code does not really care of the naming convention as long as it is 5 digits long at the maximum and that strong links start with the letter "S".

Each link needs to be associated with a (time-dependent) property and failure. During the development of CPLOAS2 it was decided to represent uncertainty over these two quantities as a multiplier toward a nominal value. The advantage of such technique is that the uncertainty does not change at each time-step which makes easier to define (one does not have to specify the parameters of the distribution at each time step) and faster to calculate.

However, it is more likely that user will have the uncertainty directly defined toward the failure criterion of a link (or toward the property of this link). In order to use CPLOAS, it is necessary to dissociate the uncertainty from the nominal value. Such decomposition is not unique, so we propose in Table 3 a convention to select a nominal value. Note that the distribution type (i.e., uniform, log-uniform, normal, log-normal, triangular, log-triangular) is not affected by such transformation.

If the nominal value is constant, then the user has to set the reference column of the nominal value (see Fig. 1 columns b and g) to zero and enter the constant value in the tpf file. If the nominal value is estimated as a function of another parameter, then a negative number is used (the absolute value representing the column that has to be used as reference) and the piecewise linear relation between the reference and nominal value is defined in the tpf file.

Table 3: proposed convention to decompose failure (resp. property) uncertainty into nominal failure (resp. property) and uncertainty factor

Uncertainty over failure (resp. property)		*Uncertainty over beta (resp. alpha)*	
Distribution	**Q : (failure)**	*Q nominal* *(to be written in tpf file if constant)*	*Beta*
Uniform	q_{min} : minimum q_{max} : maximum	$\bar{q} = 0.5(q_{min} + q_{max})$	$\beta_{min} = q_{min}/\bar{q}$ $\beta_{max} = q_{max}/\bar{q}$
Log-uniform	q_{min} : minimum q_{max} : maximum	$\bar{q} = \frac{(q_{max} - q_{min})}{\ln(q_{max}) - \ln(q_{min})}$	$\beta_{min} = q_{min}/\bar{q}$ $\beta_{max} = q_{max}/\bar{q}$
Normal	q_{μ} : mean q_{σ} : standard deviation q_{α} : truncation	$\bar{q} = q_{\mu}$	$\beta_{\mu} = 1$ $\beta_{\sigma} = q_{\sigma}/\bar{q}$ $\beta_{\alpha} = q_{\alpha}$
Lognormal	q_{μ} : mean of log q_{σ} : standard deviation of log q_{α} : truncation	$\bar{q} = e^{q_{\mu}}$	$\beta_{\mu} = 0$ $\beta_{\sigma} = q_{\sigma}$ $\beta_{\alpha} = q_{\alpha}$
Triangular	q_{min} : minimum q_{mod} : mode q_{max} : maximum	$\bar{q} = q_{mod}$	$\beta_{min} = q_{min}/\bar{q}$ $\beta_{mode} = 1$ $\beta_{max} = q_{max}/\bar{q}$
Log-triangular	q_{min} : minimum q_{mod} : mode q_{max} : maximum	$\bar{q} = q_{mod}$	$\beta_{min} = q_{min}/\bar{q}$ $\beta_{mode} = 1$ $\beta_{max} = q_{max}/\bar{q}$

The steps required to create the *CPLOAS_link.txt* file are summarized below in Fig. 17.

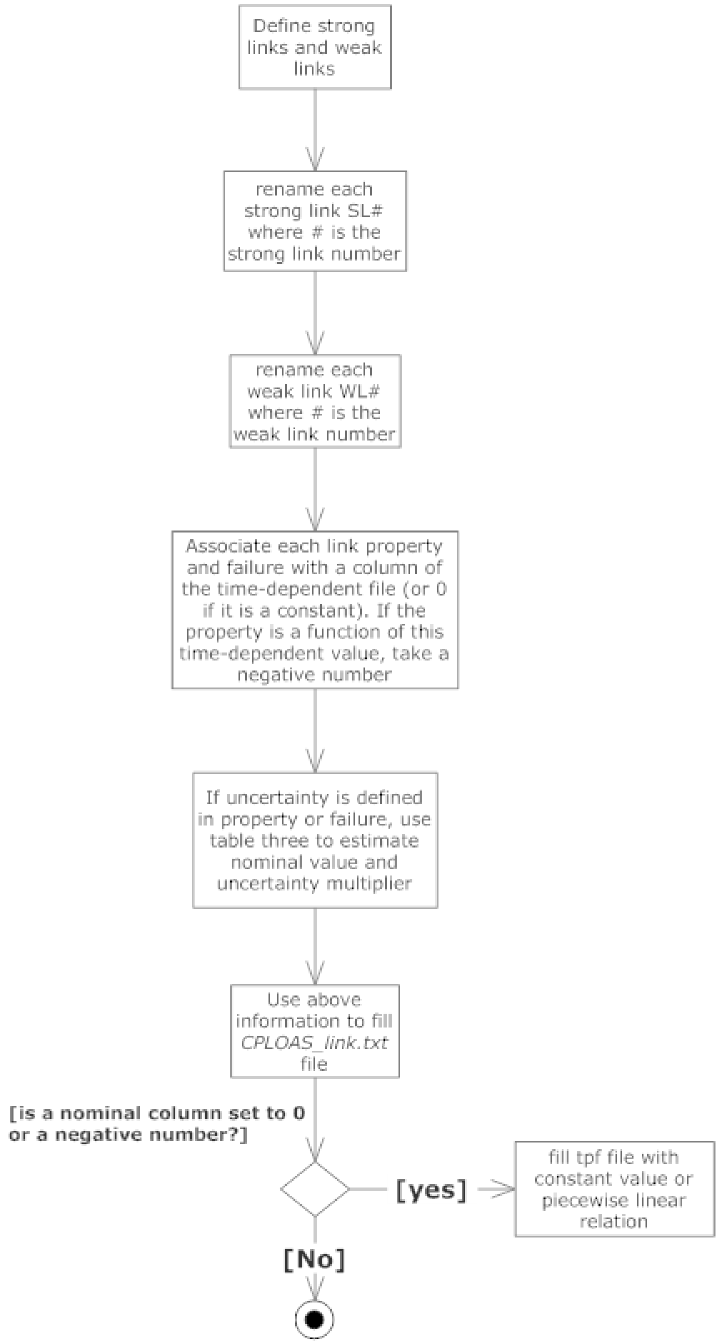


Fig. 17 Flow chart describing the steps to create *CPLOAS_link.txt* file

5.3 Definition of the tpf file

The tpf file is a text file whose name matches the analysis name specified in CPLOAS_Parameters file. The extension used is .tpf (e.g. CPCDF.tpf). The file is similar to the one defined for the previous version of CPLOAS, tpf meaning "temperature-pressure file". It was initially used to estimate failure pressure as a function of temperature via a piecewise linear approximation.

In CPLOAS2, it can be used to apply piece-wise linear transformation to any property. It can be used to design either the property or the failure but not both. It is also used to set up the constant value for a non time-dependent failure or property. Fig. 3 shows an example on how to construct such file. The file needs to exist to run the code; even if no transformation is used (it can be empty).

5.4 Definition of Circuits and creation of *CPLOAS_circuit.txt*

Once the links are defined, the following step is to define the circuits, which represent the way the links affect each other. Each circuit is defined as a collection of weak links and strong links with at least one weak link and one strong link.

The naming convention used and recommended was to use “C” followed by a number (of up to 4 digits) for simple circuit (one weak link and one strong link) and “P” followed by a number (of up to 4 digits) for a more complex circuit or a collection of circuits (called pattern).

The second information that needs to be associated to each circuit or pattern is the number informing of the circuit or pattern type. The meaning of such number is given in

Table 4. In orange are the classical 4 circuit types defined in Table 1. Number 0 (resp. -1) is used when circuits are **independent** and LOAS is obtained if ALL circuits (resp. ANY circuit) fails. Number 10 is used for an hard-wire example presented in Table 6 of Ref. [8].

Table 4: circuit type number and meaning

Circuit type	Description
-1	Failure if failure of **ANY** circuit (OR)
0	Failure if failure of **ALL** circuits (AND)
1	Failure of **all SLs** before **any WL**
2	Failure of **any SL** before **any WL**
3	Failure of **all SLs** before **all WLs**
4	Failure of **any SL** before **all WLs**
10	Hardwire example - Table 6 of Ref. [8]

Once the circuit (or pattern) type is defined, the number of links constituting the circuit (or circuit constituting the pattern) need to be entered, followed by a list of links (or circuit) names. For a circuit definition, the links names have to match the ones defined in *CPLOAS_link.txt*. For a pattern name, the circuits names have to match the ones defined in the circuits above (**the order is important**. One cannot define a pattern of two circuits prior to defining those two circuits).

Note that it is valid to create a pattern of patterns (for instance P1 is composed of C1 and C2, P2 is composed of C3 and C3, then P3 is composed of P1 and P2)

The steps required to create the *CPLOAS_circuit.txt* file are summarized below in Fig. 18

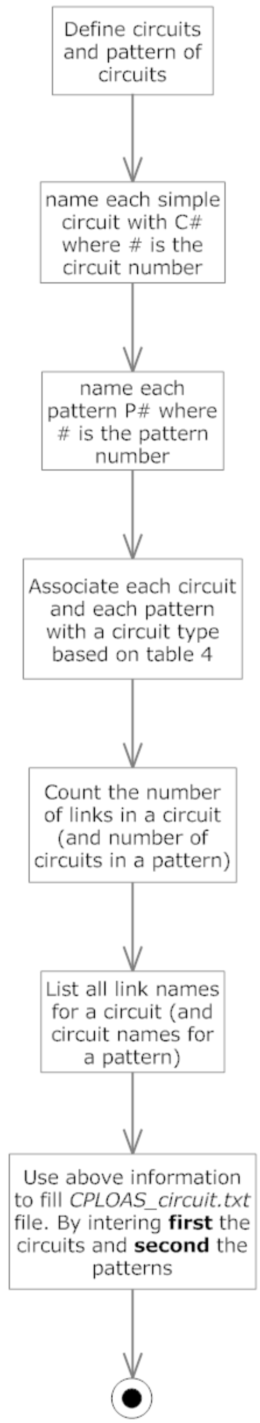


Fig. 18: Flow chart describing the steps to create *CPLOAS_circuit.txt* file

5.5 Definition of Analysis properties and creation of *CPLOAS_parameters.txt*

Once links and circuits/patterns are defined, the last step is to set the parameters of the analysis. This operation is done via the *CPLOAS_parameters.txt* file.

The first information asked is the analysis name. This name should match the one used for the time-dependent property and transformation files.

The next two numbers are discretization steps used to generate first the CDF of failure time for each link and second the calculation of LOAS for each circuit/pattern. The bigger these numbers are, the more accurate the solution will be, but it will require a longer time for the code to run. In our analyses, we used 20000 for the number of steps for the discretization to calculate the CDFs and 10000 for the quadrature discretization used to calculate LOAS and these numbers seem to work pretty well.

The following number let the user the possibility to run Monte Carlo or Importance sampling in order to check the quadrature results. If this number is set to 0, no Monte Carlo or Importance sampling technique will be used. If it is set to 1, then 2 Monte Carlo techniques (one using the CDF and another not) are performed. If it is set to 2, then 2 importance techniques (equivalent to the 2 Monte Carlo ones but using importance sampling) are performed. With the number set to 3, all four techniques are performed.

In the test we generated to verify and validate the models, the quadrature and 3 of the sampling techniques match pretty well (and match theoretical results). The fourth sampling technique (importance sampling NOT using the quadrature CDF), results tend to be not as good when the number of links involved is large compared to the probability of LOAS estimated. A rule of thumb is to consider this fourth technique when probability of LOAS ~ $10^{-nblinks}$ where *nblinks* represents the number of links involved in the calculation of LOAS. But as long as results have been confirmed once, it is perfectly valid not to run all sampling techniques and only run the quadrature approach. The quadrature approach takes usually 2 minutes while the Monte Carlo approaches can take up to 15 minutes or more.

The next two options control whether time margin and environmental margins calculations are included (values set to 1) or not (values set to 0). These two calculations are **NOT** done with a quadrature approach but rather in a sampling based technique.

The next number corresponds to the random seed. The random seed is used as a starting point by the (pseudo) random number generator and allow to repeat the same sequence of random number when needed. Any integer can be used and the use of close integer (for instance 32 and 33) will not lead to close random number sequences.

The importance sampling and Monte Carlo techniques starting with the quadrature CDF calculation use a two-steps procedure. First each CDF is sampled to create a succession of time of failure. Then the different failure times considered for each link are combined. The two following numbers in *CPLOAS_parameters.txt* are setting the sample size used for each of the procedure. The effect of changing the sample sizes is illustrated in section 4. In our tests we fixed the first number to 10,000 and the second to 100,000. It may make more sense to use the same sample size for both numbers (for instance 50,000) which we recommend. While it depends on the memory available on the computer used to run the calculation, a sample of size 5,000,000 or more lead to memory allocation failure on the machine the tests were performed.

The second set of importance and Monte Carlo technique not using the quadrature CDF are only using the second random number to calculate failure.

The second random number is also the one used to estimate time and environmental margin.

A comment line is used then to separate the parameters for an aleatory only analysis to an analysis that include epistemic uncertainty (i.e. when multiple time-dependent histories are generated).

The first number read after this comment line correspond to the espistemic sample size. **If this number is set to 1 then only aleatory calculation is performed and everything else is ignored.**

If this number is not set to 1 then **the code is expecting to have as many as time histories of properties as the epistemic sample size.**

The second number is an indicator function testing whether there will be only a single tpf file or several. If the same transformations (and constants) are used, then a single tpf file can be used and this number needs to be set to 0. If they vary for each realization then this number is set to 1 and as many as tpf files are required as the epistemic sample size.

The next two sets of three parameters allow the user to change uncertainty distribution information for a given link, at each realization (for instance if at each realization the maximum and minimum of the uncertainty factor were changing). The first three values look at the uncertainty factor on the properties (alpha values) and the next three on the uncertainty factor on the failures (beta values).
For each set, the first number indicate how many links may have varying distribution. If it is set to 0 then the next two lines will be ignored. If it is set to 1 or more, then the list of links in consideration need to be written (and match what is in *CPLOAS_link.txt*) **all in the same line**. The maximum data size needs to be indicated in the following line.

The steps required to create the *CPLOAS_circuit.txt* file are summarized below in Fig. 19

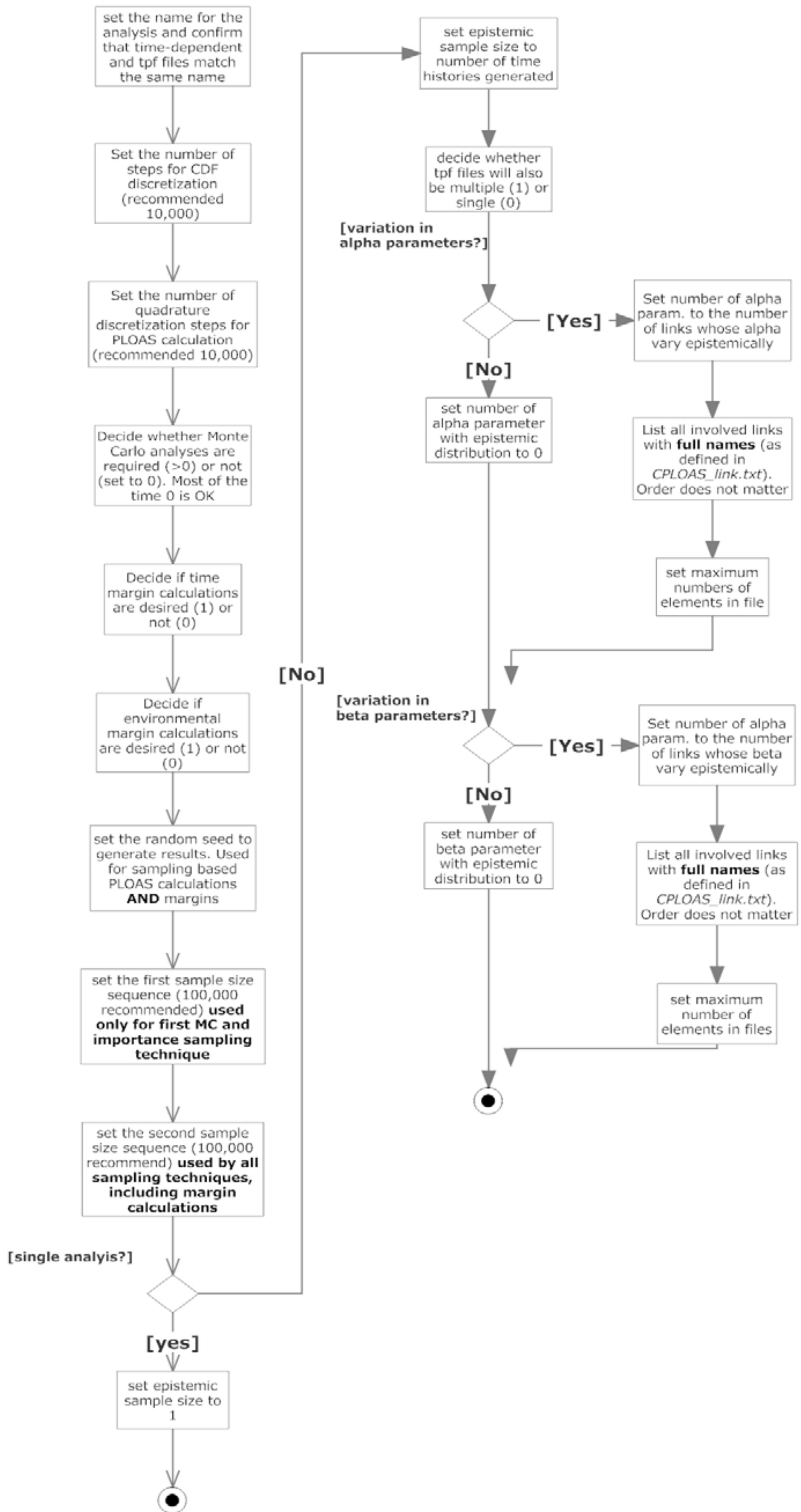


Fig. 19: Flow chart describing the steps to create *CPLOAS_parameters.txt* file

6 TEST CASES FOR CPLOAS_2

This section presents a set of test cases performed to verify the correctness of the calculation performed and to also show at the same time how to construct inputs files in order to use CPLOAS_2.

6.1 Analytical test 1

6.1.1 Description of the test case

The first test case is based on the set of test problems presented in Sect. 6 of Ref. [8] and involves a system with 2 WLs and 2 SLs with the same nominal properties and failure values assigned to all links (Fig. 20). Specifically, the nominal properties and failure values for the links are defined by $\overline{p}(t) = 100 + 3t$ and $\overline{q}(t) = 600 - 2t$, respectively, for $0 \le t \le 200$ min (Fig. 20a). The distributions for the alpha values characterizing aleatory uncertainty in the nominal link properties are uniform on [0.85, 1.15], and the distributions for the beta values characterizing aleatory uncertainty in the nominal link failure values are triangular on [0.9, 1.1] with a mode of 0.0. The distributions of time-dependent link properties and failure values that result from the preceding distributions for alpha values and beta values are shown in Fig. 20b. In turn, these distributions result in the same CDF for link failure time for each of the four links (Fig. 20c).

Four possible configurations of WL-SL systems are defined in Table 1 and designated Case 1, 2, 3 and 4. In addition, test values for PLOAS are also given that result when all links are assigned the same properties and failure values. The test values for PLOAS for Cases 1, 2, 3 and 4 for 2 WLs and 2 SLs are 1/6, 1/2 , 1/2 and 5/6, respectively, as illustrated in Table 3 of Ref. [8].

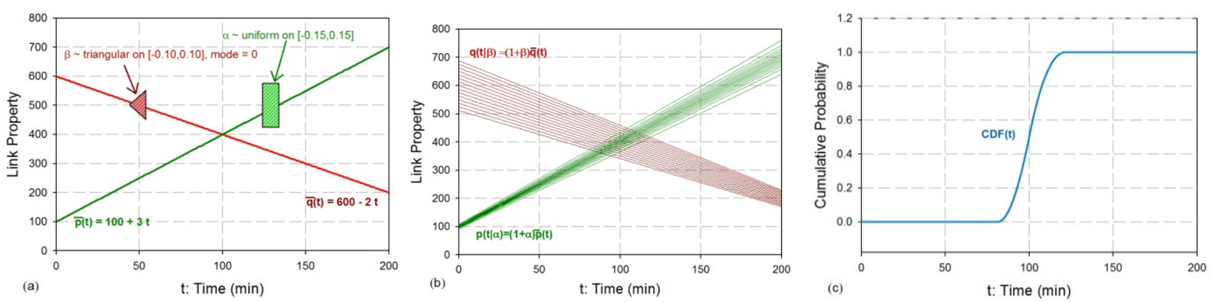
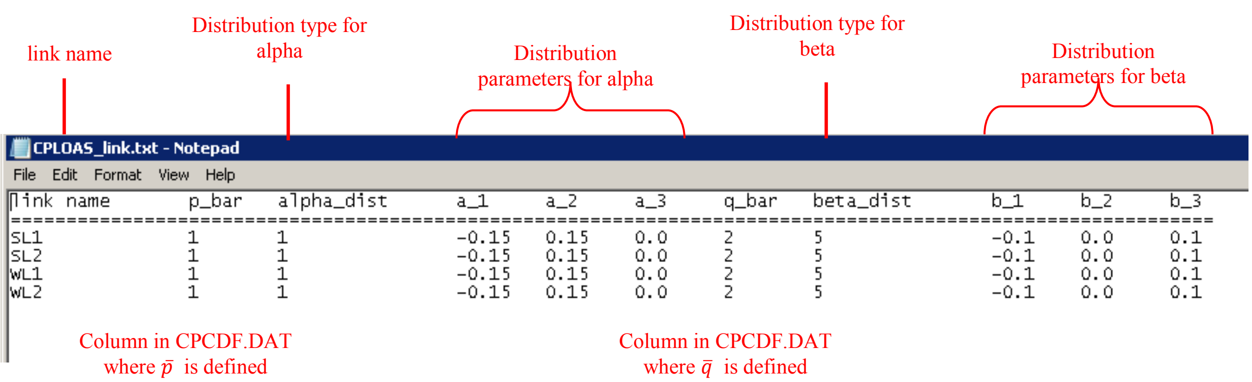


Fig. 20 Link properties for illustration of verification tests: (a) base physical property $\overline{p}(t)$, base failure property $\overline{q}(t)$, and distributions for aleatory variables α and β, (b) physical properties $p(t|\alpha) = \alpha\overline{p}(t)$ and failure properties $q(t|\beta) = \beta\overline{q}(t)$ generated with random samples of size 100 from the distributions for α and β, and (c) cumulative distribution $CDF(t)$ for link failure time.

6.1.2 Construction of the test case

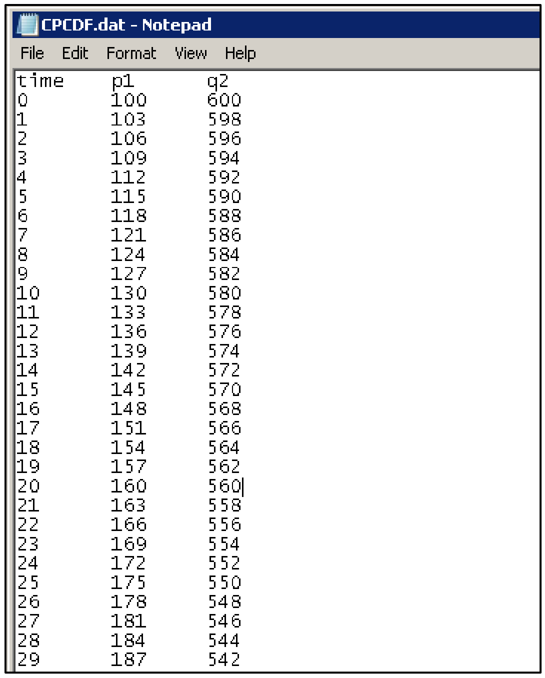
The link properties are defined in the input file **CPLOAS_link.txt** (Fig. 21). Each SL and WL is assigned the same properties in this test case (see Fig. 20). As indicated, the nominal properties are read from column 1 of **CPCDF.dat** (the time column is not counted; see Fig. 22). The alpha distributions are uniform (designator associated with uniform distribution = 1). The parameters of the uniform distribution are −0.15 (minimum), 0.15(maximum) and 0.0 (placeholder; i.e., the uniform distribution is defined by only 2 parameters). The nominal failure values are read from column 2 of **CPCDF.dat** (again, time column is not counted). The beta distributions are triangular (designator associated with triangular distribution = 5), which minimum, mode and maximum equal to −0.1, 0.0 and 0.1, respectively.

The **CPCDF.dat** file for this example contains three columns. The first column contains the times (201 values from 0 to 200 corresponding to one minute time steps). The second and third columns contain the nominal property and failure values, respectively, at the corresponding time-step (see Fig. 22).




```
link name      p_bar   alpha_dist    a_1     a_2     a_3     q_bar   beta_dist     b_1     b_2     b_3
=====================================================================================================
SL1            1       1             -0.15   0.15    0.0     2       5             -0.1    0.0     0.1
SL2            1       1             -0.15   0.15    0.0     2       5             -0.1    0.0     0.1
WL1            1       1             -0.15   0.15    0.0     2       5             -0.1    0.0     0.1
WL2            1       1             -0.15   0.15    0.0     2       5             -0.1    0.0     0.1
```

Fig. 21 Input file **CPLOAS_link.txt** for test case 1 (see Fig. 1 for additional discussion).

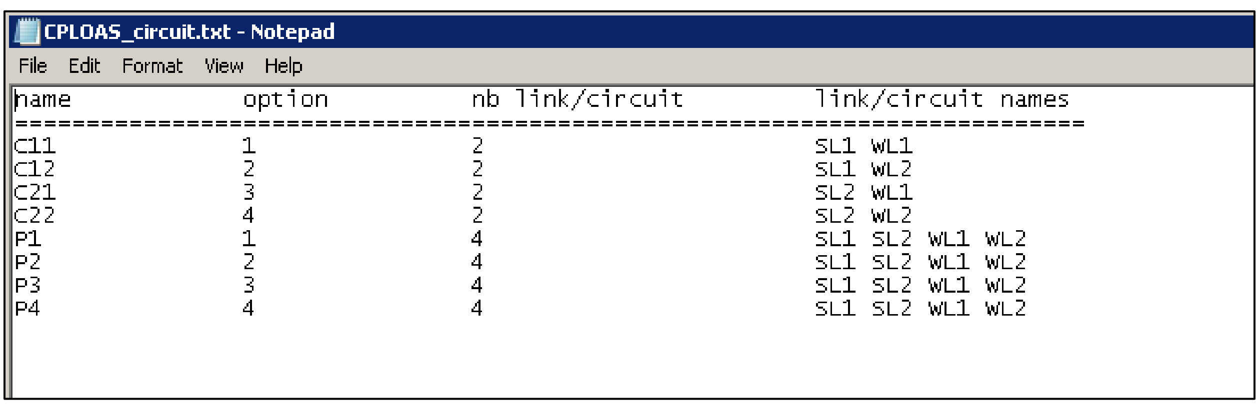

CPCDF.dat - Notepad

File Edit Format View Help

time	p1	q2
0	100	600
1	103	598
2	106	596
3	109	594
4	112	592
5	115	590
6	118	588
7	121	586
8	124	584
9	127	582
10	130	580
11	133	578
12	136	576
13	139	574
14	142	572
15	145	570
16	148	568
17	151	566
18	154	564
19	157	562
20	160	560
21	163	558
22	166	556
23	169	554
24	172	552
25	175	550
26	178	548
27	181	546
28	184	544
29	187	542

Fig. 22 First 29 time steps of input file **CPCDF.DAT** for test case 1 (see Fig. 2 for additional discussion).

Circuits for this example are defined in the file **CPLOAS_circuit.txt** (Fig. 23). Although it is not necessary for this test case, circuits have been defined for each possible WL-SL pair. Then, for a circuit with 2 SLs and 2 WLs, the four possible failure patterns are specified (Corresponding to the second column of the file and associated named option).


CPLOAS_circuit.txt - Notepad

File Edit Format View Help

```
name            option          nb link/circuit         link/circuit names
=========================================================================
C11             1               2                       SL1 WL1
C12             2               2                       SL1 WL2
C21             3               2                       SL2 WL1
C22             4               2                       SL2 WL2
P1              1               4                       SL1 SL2 WL1 WL2
P2              2               4                       SL1 SL2 WL1 WL2
P3              3               4                       SL1 SL2 WL1 WL2
P4              4               4                       SL1 SL2 WL1 WL2
```

Fig. 23 Input file **CPLOAS_circuit.txt** for test case 1 (see Fig. 5 for additional discussion).

The last input file that be defined is **CPLOAS_parameters.txt** (Fig. 24). In this file, the user first defines the base name for the output files (i.e., CPCDF is used in the test case). The next set of numbers (all integers) specifies the properties of the methodology used to determine PLOAS. The first two numbers are used for the quadrature and correspond to the number of steps for the CDF discretization (set to *nCDF* = 20,000) and for the quadrature discretization (set to *nQUAD*

= 10,000). The following number (i.e., *nRSEED* = 352) is used as random seed for sampling techniques. The last two numbers relate to the initial random selection of link failure times from each link failure time CDF (set to *nFT* = 1,000,000) and to the combinations of times (set to *nFTC* = 1,000,000). After a comment line, epistemic sample size is set to 1 (the test case considers only aleatory uncertainty), resulting in the following rows to be ignored except for the fourth following row where the numerical procedures to be used to determine PLOAS are specified.

```
CPLOAS_parameters.txt - Notepad
File  Edit  Format  View  Help
CPCDF                                  ! Output filename for CDF for each link
20000                                      ! number of steps for CDF discretization (= 2000 in example)
1000                                       ! number of quadrature discretization steps (= 10000 in example)
32                                        ! seed to generate random sequences
1000000                                    ! sample size for time selection for each link (= 10000 in example)
1000000                                            ! number of time combinations for MC and IMP sampling (= 100000 in
example)
=================================== COMMENT LINE=====================================================================
1                                          ! epistemic sample size (set to 1 if no epistemic)
1                                          ! failure included (0 = no epistemic on failure - 1 = epistemic on failure)
3                                          ! sampling included (0 = no sampling ; 1 = MC ; 2 = Importance ; 3 = both)
0                                          ! number of alpha parameters with epistemic distribution
SL1 WL1                                    ! list of links (the way they appear in CPLOAS_link.txt) for alpha parameters
10000                                      ! maximum data size for alpha parameters
0                                          ! number of beta parameters with epistemic distribution
WL1 SL4 SL5                                ! list of links (the way they appear in CPLOAS_link.txt) for beta parameters
1                                          ! maximum data size for beta parameters
```

Fig. 24 Input file **CPLOAS_parameters.txt** for test case 1 (see Fig. 6 for additional discussion).

6.1.3 Results for test case 1

Results obtained using three methods to determine PLOAS for four different definitions of LOAS are displayed in Table 5. With discretizations of size *nCDF* = 20,000 and *nQUAD* = 10,000, quadrature deviates from the true values for PLOAS by less than 0.5%. Importance sampling with samples of size *nFT* = *nFTC* = 1,000,000 has comparable errors, and the errors with simple random (i.e., Monte Carlo) sampling with samples of size *nFT* = *nFTC* = 1,000,000 are less than 1%. The two sampling procedures used to produce the results in Table 5 are referred to as random sampling 1 and importance sampling 1 in Sect. 4.

Table 5 Comparison of results at 200 min for test case 1

		Quadrature (20K,10K)		Monte Carlo (1M)		Importance (1M)	
Pattern type	**theoretical result**	**value**	**diff. in %**	**value**	**diff. in %**	**value**	**diff. in %**
1	0.16667	0.1662	-0.28%	0.1662	-0.28%	0.1662	-0.28%
2	0.5	0.4998	-0.04%	0.5021	0.42%	0.4997	-0.06%
3	0.5	0.4991	-0.18%	0.5030	0.60%	0.4981	-0.38%
4	0.83333	0.8336	0.03%	0.8405	0.86%	0.8308	-0.30%

6.2 Analytical test 2

6.2.1 Description of the test case

In the second test case, the problem is defined similarly to test case 1. The only change is for the distribution associated for each alpha and beta value. The alpha distributions are normal with mean of 1.0, standard deviation of 0.05 and truncation at q = 0.001. The beta distributions are normal with mean = 1.0, standard deviation = 0.005 and truncation at q = 0.001. The theoretical results for cases 1,2,3 and 4 for 2 SLs and 2 WLs stays at 1/6, 1/2 , 1/2 and 5/6. This test case indicates that truncation as well as the definition of the normal CDF are implemented correctly.

6.2.2 Construction of the test case

Only **CPLOAS_link.txt** differs from the input files used for test case 1 (Fig. 25). In test case 2, the distribution designators for alpha are changed to 3 (indicating a normal distribution), and the corresponding parameters are changed to 1.0, 0.05 and 0.001. The distributions designators for beta are also changed to 3 (indicating a normal distribution), and the corresponding parameters are changed to 1.0, 0.005 and 0.001, respectively.

```
CPLOAS_link.txt - Notepad
File  Edit  Format  View  Help
link name      p_bar    alpha_dist       a_1     a_2     a_3     q_bar   beta_dist        b_1     b_2     b_3
=============================================================================================================
SL1            1        3                0.0     0.05    0.001   2       3                -0.1    0.005   0.001
SL2            1        3                0.0     0.05    0.001   2       3                -0.1    0.005   0.001
WL1            1        3                0.0     0.05    0.001   2       3                -0.1    0.005   0.001
WL2            1        3                0.0     0.05    0.001   2       3                -0.1    0.005   0.001
```

Fig. 25 Input file **CPLOAS_link.txt** for test case 2 (see Fig. 1 for additional discussion).

6.2.3 Results for test case 2

Results using the three methods are displayed in Table 6. With discretization of size $nCDF$ = 20,000 and $nQUAD$ = 10,000, quadrature loses some accuracy compared with the results for test case 1 in Table 5 (i.e., difference with theory increased by a factor of approximately two). Importance sampling and simple random (Monte Carlo) sampling accuracies are similar to the results for test case 1 in Table 5.

Table 6 Comparison of results at 200 min for test case 2

		quadrature (20K,10K)		Monte Carlo (1M)		Importance (1M)		quadrature (50K,50K)	
Pattern type	**theoretical result**	**value**	**diff. in %**	**value**	**diff. in %**	**value**	**diff. in %**	**value**	**diff. in %**
1	0.16667	0.1657	-0.58%	0.1662	-0.28%	0.1662	-0.28%	0.1665	-0.10%
2	0.5	0.4996	-0.08%	0.5021	0.42%	0.4997	-0.06%	0.4999	-0.02%
3	0.5	0.4983	-0.34%	0.503	0.60%	0.4981	-0.38%	0.4997	-0.06%
4	0.83333	0.8337	0.04%	0.8405	0.86%	0.8308	-0.30%	0.8334	0.01%

Increasing the number of discretization points from (*nCDF* = 20K, *nQUAD* = 10K) to (*nCDF* = 50K, *nQUAD* = 50K) confirms that the quadratic solution still converges. Estimated results are then within 0.1% of the theoretical values.

6.3 Analytical test 3

6.3.1 Description of the test case

The third test case is a little more complex as it considers the time-dependent evolution of both a CDF for cumulative link failure and the probability of loss of assured safety (i.e., PLOAS). For this test case, only one WL and one SL are considered. Their nominal property and failure values are assumed to be the same. Specifically, the nominal property and failure values over time t are defined by

$$\bar{p}(t) = 100 + 4t \text{ and } \bar{q}(t) = 600 - t, \tag{6.1}$$

respectively, for $0 \leq t \leq 200$ min, and the distributions for the corresponding alpha and beta values are assumed to be uniform on [0.9, 1.1].

Evaluation of the representation *CDF*(t) for the cumulative failure probability defined in Eq. (2.12) of Ref. [8] for a link with the properties defined in conjunction with Eq. (5.1) produces the result

$$\begin{aligned} CDF(t) &= 0 \text{ for } t < 81.1 \text{ min} \\ &= 15.125\left[\bar{p}(t)/\bar{q}(t)\right] + 10.125\left[\bar{q}(t)/\bar{p}(t)\right] - 24.75 \text{ for } 81.1 \text{ min } \leq t < 100 \text{ min} \\ &= -10.125\left[\bar{p}(t)/\bar{q}(t)\right] - 15.125\left[\bar{q}(t)/\bar{p}(t)\right] + 25.75 \text{ for } 100 \text{ min } \leq t < 121.2 \text{ min} \\ &= 1.0 \text{ for } 121.2 \text{ min } \leq t. \end{aligned} \tag{6.2}$$

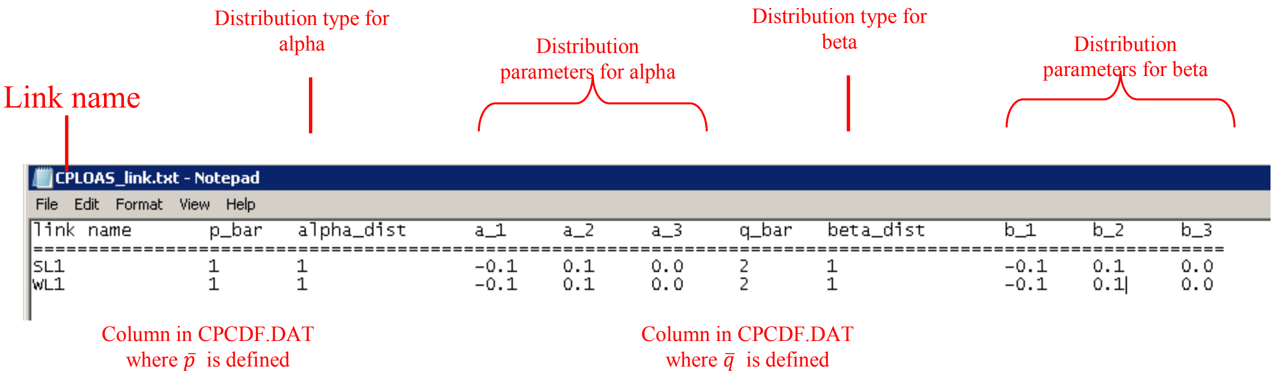
In turn, the probability for the SL to fail before the WL when both links have the time-dependent failure probability in Eq. (5.2) can be easily estimated numerically. For one WL and one SL, PLOAS as function of time is defined by Case 1 in Table 1 and represented by $P_1(t)$ as stated below:

$$\begin{aligned} P_1(t) &= \int_0^t \left[1 - CDF(t)\right] \mathrm{d}CDF(t) \\ &\cong \sum_{i=1}^{n} \left[1 - CDF(t_i)\right]\left[CDF(t_i) - CDF(t_{i-1})\right], \end{aligned} \tag{6.3}$$

where $0 = t_0 < t_1 < \cdots < t_n = t$ is a subdivision of [0, t].

6.3.2 Construction of the test case

The link properties are defined in **CPLOAS_link.txt** (Fig. 26). Both links have the same properties in this test case. The nominal property values are defined in column 1 of the file **CPCDF.dat** (the time column is not counted; see Fig. 27). The alpha distributions are uniform (code associated with uniform = 1). The parameters of the uniform distributions are 0.9 (minimum), 1.1 (maximum) and 0.0 (placeholder). A uniform distribution uses only 2 parameters. The nominal failure values are defined in column 2 of **CPCDF.dat** (again, the time column is not counted). The beta distributions are uniform (code associated with uniform = 1), with minimum and maximum equal to 0.9 and 1.1, respectively, and a placeholder value of 0.0 used for the third parameter.




```
link name     p_bar   alpha_dist      a_1     a_2     a_3     q_bar   beta_dist       b_1     b_2     b_3
========================================================================================================
SL1           1       1               -0.1    0.1     0.0     2       1               -0.1    0.1     0.0
WL1           1       1               -0.1    0.1     0.0     2       1               -0.1    0.1     0.0
```

Fig. 26 Input file **CPLOAS_link.txt** for test case 3 (see Fig. 1 for additional discussion).

The input file **CPCDF.dat** is constructed similarly to the first two test cases with one column representing the nominal property values and a second column representing the nominal failure value (Fig. 27).

Circuits and patterns are defined in the input file **CPLOAS_circuit.txt** (Fig. 28). A single circuit involving the two links is defined, which is the only possibility when only one WL and one SL is defined. The option is set to 1, although any of the four options in Table 1 involves the same WL-SL configuration when only one WL and one SL is under consideration.

The last of the parameter file to be defined is **CPLOAS_parameters.txt**. The options used are the same as for test cases 1 and 2 and can be seen in Fig. 24.

CPCDF.dat - Notepad

File Edit Format View Help

```
time    p1     q2
0       100    600
1       104    599
2       108    598
3       112    597
4       116    596
5       120    595
6       124    594
7       128    593
8       132    592
9       136    591
10      140    590
11      144    589
12      148    588
13      152    587
14      156    586
15      160    585
16      164    584
17      168    583
18      172    582
19      176    581
20      180    580
21      184    579
22      188    578
23      192    577
24      196    576
25      200    575
26      204    574
27      208    573
28      212    572
29      216    571
30      220    570
31      224    569
32      228    568
33      232    567
34      236    566
```

Fig. 27 First 34 time steps of input file **CPCDF.dat** for test case 3 (see Fig. 2 for additional discussion).

CPLOAS_circuit.txt - Notepad

File Edit Format View Help

```
name            option          nb link/circuit         link/circuit names
============================================================================
C11             1               2                       SL1 WL1
```

Fig. 28 Input file **CPLOAS_circuit.txt** for test case 3 (see Fig. 5 for additional discussion).

6.3.3 Results for test case 3

Initially, results obtained numerically with CPLOAS_2 for the failure time CDF *CDF*(t) for the two links are compared with the results obtained from a direct evaluation of this CDF as defined in Eq. (6.2) As shown in Fig. 29, the two evaluations of *CDF*(t) are visually indistinguishable.

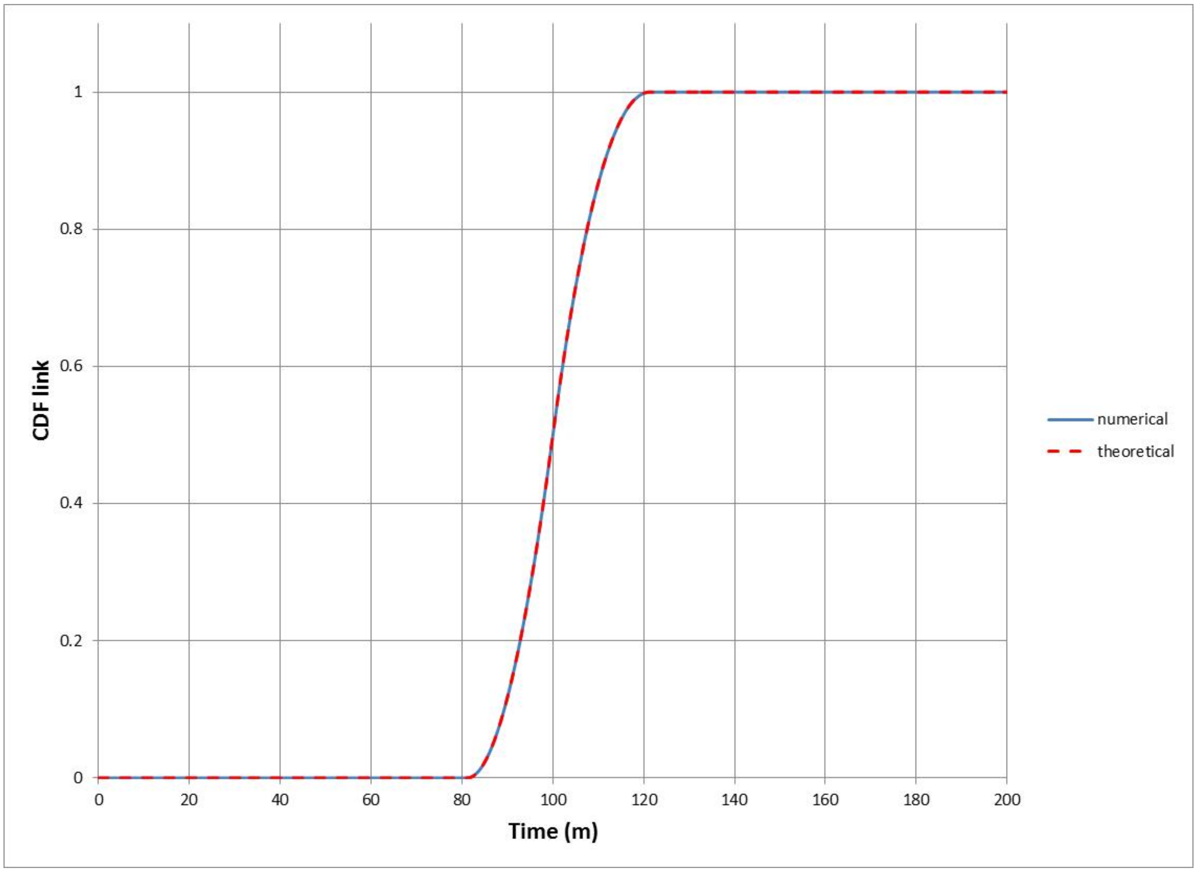


Fig. 29 Comparison between theoretical CDF (red dash) and estimate from CPLOAS2 (blue line) for test case 3.

Next, results obtained numerically with CPLOAS_2 for the PLOAS values $P_1(t)$ for the two links are compared with results obtained in an independently implemented evaluation of $P_1(t)$ with the approximation in Eq. (6.3) and the closed form representation for *CDF*(t) in Eq. (6.2) As shown in Fig. 30, the two evaluations of $P_1(t)$ are visually indistinguishable.

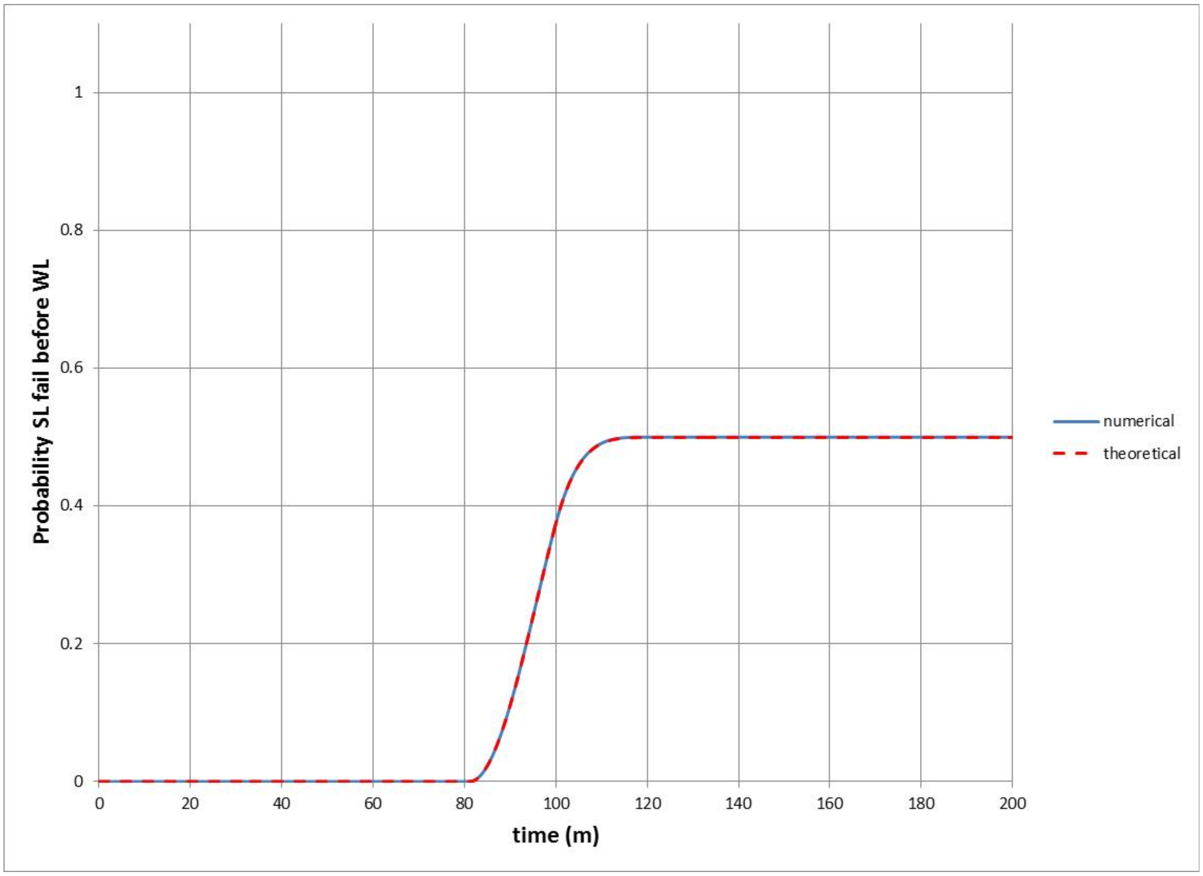


Fig. 30 Comparison of probability of SL failing before WL for test case 3.

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APPENDIX A: SUMMARY OF NUMERICAL PROCEDURES USED IN CPLOAS_2 TO CALCULATE PROBABILITY OF LOSS OF ASSURED SAFETY

A.1 Introduction

This appendix provides a technical summary of the numerical procedures being tested in this report for the calculation of PLOAS. Specifically, the following numerical procedures are described: (i) quadrature procedure (Sect. A.2), (ii) random sampling procedure 1 (MC1) (Sect. A.3), (iii) random sampling procedure 2 (MC2) (Sect. A.4), (iv) importance sampling procedure 1 (IMP1) (Sect. A.4), and (v) importance sampling procedure 2 (IMP2) (Sect. A.5). Additional information and illustrations associated with these procedures are available in Ref. [1].

A.2 Quadrature Procedure

The defining integrals for PLOAS implemented in CPLOAS_2 are defined as shown in Table A.1 with $CDF_{WL,j}(\tau)$ and $CDF_{SL,k}(\tau)$ representing the cumulative distribution functions (CDFs) for WL failure time and SL failure time, respectively. As described in Sect. 2 of Ref. [1], the failure time CDF for a single WL or SL is based on the following assumed properties of that link for a time interval $t_{mn} \le t \le t_{mx}$:

$$\bar{p}(t) = \text{nondecreasing function defining nominal link property for } t_{mn} \le t \le t_{mx}, \tag{A.1}$$

$$\bar{q}(t) = \begin{array}{l}\text{nonincreasing function defining nominal failure value for link property} \\ \text{for } t_{mn} \le t \le t_{mx},\end{array} \tag{A.2}$$

$$d_\alpha(\alpha) = \begin{array}{l}\text{density function for variable } \alpha \text{ used to characterize aleatory uncertainty} \\ \text{in link property,}\end{array} \tag{A.3}$$

$$d_\beta(\beta) = \begin{array}{l}\text{density function for variable } \beta \text{ used to characterize aleatory uncertainty} \\ \text{in link failure value,}\end{array} \tag{A.4}$$

$$p(t \mid \alpha) = \alpha\bar{p}(t) = \text{ link property for } t_{mn} \le t \le t_{mx} \text{ given } \alpha, \tag{A.5}$$

and

$$q(t \mid \beta) = \beta\bar{q}(t) = \text{ link failure value for } t_{mn} \le t \le t_{mx} \text{ given } \beta. \tag{A.6}$$

Further, $d_\alpha(\alpha)$ and $d_\beta(\beta)$ are assumed to be defined on intervals $[\alpha_{mn}, \alpha_{mx}]$ and $[\beta_{mn}, \beta_{mx}]$ and to equal zero outside these intervals.

Table A.1 Representation of time-dependent values $pF_i(t)$, i = 1, 2, 3, 4, for PLOAS and associated verification tests for alternate definitions of LOAS for WL/SL Systems with (i) nWL WLs and nSL SLs and (ii) independent distributions for link failure time ([2], Table 10)

Case 1: Failure of all SLs before failure of any WL (Eqs. (2.1) and (2.5), Ref. [3])

$$pF_1(t) = \sum_{k=1}^{nSL} \left(\int_0^t \left\{ \prod_{\substack{l=1 \\ l \neq k}}^{nSL} CDF_{SL,l}(\tau) \right\} \left\{ \prod_{j=1}^{nWL} \left[1 - CDF_{WL,j}(\tau) \right] \right\} \mathrm{d}CDF_{SL,k}(\tau) \right)$$

Verification test: $pF_1(\infty) = nWL!\,nSL!/(nWL + nSL)!$

Case 2: Failure of any SL before failure of any WL (Eqs. (3.1) and (3.4), Ref. [3])

$$pF_2(t) = \sum_{k=1}^{nSL} \left(\int_0^t \left\{ \prod_{\substack{l=1 \\ l \neq k}}^{nSL} \left[1 - CDF_{SL,l}(\tau) \right] \right\} \left\{ \prod_{j=1}^{nWL} \left[1 - CDF_{WL,j}(\tau) \right] \right\} \mathrm{d}CDF_{SL,k}(\tau) \right)$$

Verification test: $pF_2(\infty) = nSL/(nWL + nSL)$

Case 3: Failure of all SLs before failure of all WLs (Eqs. (4.1) and (4.4), Ref. [3])

$$pF_3(t) = \sum_{k=1}^{nSL} \left(\int_0^t \left\{ \prod_{\substack{l=1 \\ l \neq k}}^{nSL} CDF_{SL,l}(\tau) \right\} \left\{ 1 - \prod_{j=1}^{nWL} CDF_{WL,j}(\tau) \right\} \mathrm{d}CDF_{SL,k}(\tau) \right)$$

Verification test: $pF_3(\infty) = nWL/(nWL + nSL)$

Case 4: Failure of any SL before failure of all WLs (Eqs. (5.1) and (5.4), Ref. [3])

$$pF_4(t) = \sum_{k=1}^{nSL} \left(\int_0^t \left\{ \prod_{\substack{l=1 \\ l \neq k}}^{nSL} \left[1 - CDF_{SL,l}(\tau) \right] \right\} \left\{ 1 - \prod_{j=1}^{nWL} CDF_{WL,j}(\tau) \right\} \mathrm{d}CDF_{SL,k}(\tau) \right)$$

Verification test: $pF_4(\infty) = 1 - \left[nWL!\,nSL!/(nWL + nSL)! \right]$

Once $CDF(t)$ and $CCDF(t) = 1 - CDF(t)$ are evaluated for individual links, the representations for PLOAS in Table A.1 can be numerically evaluated with a quadrature procedure. Specifically, the probability $pF_1(t)$ for the failure all SLs before the failure of any WL defined as Case 1 in Table A.1 is approximated in CPLOAS_2 by

$$
\begin{aligned}
pF_1(t) &\cong \sum_{k=1}^{nSL}\left(\sum_{i=1}^{n}\left\{\prod_{l=1,l\neq k}^{nSL} CDF_{SL,l}\left(t_{i-1}\right)\right\}\right. \\
&\quad \left.\times\left\{\prod_{j=1}^{nWL}\left[1-CDF_{WL,j}\left(t_i\right)\right]\right\}\left\{CDF_{SL,k}\left(t_i\right)-CDF_{SL,k}\left(t_{i-1}\right)\right\}\right) \\
&= \sum_{k=1}^{nSL}\left(\sum_{i=1}^{n}\left\{\prod_{l=1,l\neq k}^{nSL} CDF_{SL,l}\left(t_{i-1}\right)\right\}\right. \\
&\quad \left.\times\left\{\prod_{j=1}^{nWL}\left[CCDF_{WL,j}\left(t_i\right)\right]\right\}\left\{CDF_{SL,k}\left(t_i\right)-CDF_{SL,k}\left(t_{i-1}\right)\right\}\right)
\end{aligned}
\tag{A.7}
$$

for subdivisions $0 = t_0 < t_1 < \ldots < t_n = t$ of $[0, t]$. As shown below, similar approximations are also used in CPLOAS_2 for the other three failure cases defined in Table A.1. In the preceding approximation for $pF_1(t)$, left and right evaluations are indicated for SLs (i.e., $CDF_{SL,l}(t_{i-1})$ and $CCDF_{SL,l}(t_{i-1})$) and WLs (i.e., $CDF_{WL,j}(t_i)$ and $CCDF_{WL,j}(t_i)$), respectively, as the underlying assumption is that all SLs except for SL k have failed before time t_{i-1} and all WLs fail after time t_i. If the CDFs and CCDFs are continuous in time, this specification of evaluation times does not affect the limiting value for $pF_1(t)$ as Δt_i goes to zero.

Similarly, the representations $pF_2(t)$, $pF_3(t)$ and $pF_4(t)$ for PLOAS in table A.1 are approximated in CPLOAS_2 by

$$
\begin{aligned}
pF_2(t) &\cong \sum_{k=1}^{nSL}\left(\sum_{i=1}^{n}\left\{\prod_{l=1,l\neq k}^{nSL}\left[1-CDF_{SL,l}\left(t_{i-1}\right)\right]\right\}\right. \\
&\qquad \left.\times\left\{\prod_{j=1}^{nWL}\left[1-CDF_{WL,j}\left(t_i\right)\right]\right\}\left\{CDF_{SL,k}\left(t_i\right)-CDF_{SL,k}\left(t_{i-1}\right)\right\}\right) \\
&= \sum_{k=1}^{nSL}\left(\sum_{i=1}^{n}\left\{\prod_{l=1,l\neq k}^{nSL} CCDF_{SL,l}\left(t_{i-1}\right)\right\}\right. \\
&\qquad \left.\times\left\{\prod_{j=1}^{nWL} CCDF_{WL,j}\left(t_i\right)\right\}\left\{CDF_{SL,k}\left(t_i\right)-CDF_{SL,k}\left(t_{i-1}\right)\right\}\right),
\end{aligned}
\tag{A.8}
$$

$$
\begin{aligned}
pF_3(t) &\cong \sum_{k=1}^{nSL}\left(\sum_{i=1}^{n}\left\{\prod_{l=1,l\neq k}^{nSL} CDF_{SL,l}\left(t_{i-1}\right)\right\}\right. \\
&\qquad \left.\times\left\{1-\prod_{j=1}^{nWL} CDF_{WL,j}\left(t_i\right)\right\}\left\{CDF_{SL,k}\left(t_i\right)-CDF_{SL,k}\left(t_{i-1}\right)\right\}\right)
\end{aligned}
\tag{A.9}
$$

and

$$
\begin{aligned}
pF_4(t) &\cong \sum_{k=1}^{nSL}\left(\sum_{i=1}^{n}\left\{\prod_{l=1,l\neq k}^{nSL}\left[1-CDF_{SL,l}\left(t_{i-1}\right)\right]\right\}\right. \\
&\qquad \left.\times\left\{1-\prod_{j=1}^{nWL} CDF_{WL,j}\left(t_i\right)\right\}\left\{CDF_{SL,k}\left(t_i\right)-CDF_{SL,k}\left(t_{i-1}\right)\right\}\right) \\
&= \sum_{k=1}^{nSL}\left(\sum_{i=1}^{n}\left\{\prod_{l=1,l\neq k}^{nSL} CCDF_{SL,l}\left(t_{i-1}\right)\right\}\right. \\
&\qquad \left.\times\left\{1-\prod_{j=1}^{nWL} CDF_{WL,j}\left(t_i\right)\right\}\left\{CDF_{SL,k}\left(t_i\right)-CDF_{SL,k}\left(t_{i-1}\right)\right\}\right)
\end{aligned}
\tag{A.10}
$$

for subdivisions $0 = t_0 < t_1 < \ldots < t_n = t$ of $[0, t]$.

In the numerical evaluation of the indicated integrals in CPLOAS_2, the individual link failure time CDFs are discretized by dividing time interval under consideration into *nCDF* equally-spaced discretization steps and the integrals are approximated by dividing time interval under consideration into *nQUAD* equally-spaced discretization steps. As needed, linear interpolation is used to incorporate the link failure CDFs into the quadrature approximations to PLOAS.

A.3 Sampling Procedure 1 (MC1)

Sampling Procedure 1 (MC1) is based to estimate the expected values of functions $\delta_i(t \mid \mathbf{t})$, i = 1, 2, 3, 4, where

$$t = \text{ time at which PLOAS (i.e., } pF_i(t) \text{ in Table 1) is to be determined,} \tag{A.11}$$

$$tWL_j = \text{ time at which WL } j \text{ fails, } j = 1, 2, ..., nWL, \tag{A.12}$$

$$tSL_j = \text{ time at which SL } j \text{ fails, } j = 1, 2, ..., nSL, \tag{A.13}$$

$$\mathbf{t} = \left[tWL_1, tWL_2, ..., tWL_{nWL}, tSL_1, tSL_2, ..., tSL_{nSL} \right], \tag{A.14}$$

$$\delta_1\left(t \mid \mathbf{t}\right) = \begin{cases} 1 \text{ if } \max\left\{tSL_1, tSL_2, ..., tSL_{nSL}\right\} \le \min\left\{t, tWL_1, tWL_2, ..., tWL_{nSL}\right\} \\ 0 \text{ otherwise,} \end{cases} \tag{A.15}$$

$$\delta_2\left(t \mid \mathbf{t}\right) = \begin{cases} 1 \text{ if } \min\left\{tSL_1, tSL_2, ..., tSL_{nSL}\right\} \le \min\left\{t, tWL_1, tWL_2, ..., tWL_{nSL}\right\} \\ 0 \text{ otherwise,} \end{cases} \tag{A.16}$$

$$\delta_3\left(t \mid \mathbf{t}\right) = \begin{cases} 1 \text{ if } \max\left\{tSL_1, tSL_2, ..., tSL_{nSL}\right\} \le \min\left\{t, \max\left\{tWL_1, tWL_2, ..., tWL_{nSL}\right\}\right\} \\ 0 \text{ otherwise,} \end{cases} \tag{A.17}$$

and

$$\delta_4\left(t \mid \mathbf{t}\right) = \begin{cases} 1 \text{ if } \min\left\{tSL_1, tSL_2, ..., tSL_{nSL}\right\} \le \min\left\{t, \max\left\{tWL_1, tWL_2, ..., tWL_{nSL}\right\}\right\} \\ 0 \text{ otherwise.} \end{cases} \tag{A.18}$$

In words, $\delta_1(t \mid \mathbf{t}) = 1$ corresponds to all SLs failing before time t and also before any WL fails (i.e., Case 1 in Table A.1); $\delta_2(t \mid \mathbf{t}) = 1$ corresponds to any SL failing before time t and also before any WL fails (i.e., Case 2 in Table A.1); $\delta_3(t \mid \mathbf{t}) = 1$ corresponds to all SLs failing before time t and also before all WLs fail (i.e., Case 3 in Table A.1); and $\delta_4(t \mid \mathbf{t}) = 1$ corresponds to any SL failing before time t and also before all WLs fail (i.e., Case 4 in Table A.1). If a time interval $[t_{mn}, t_{mx}]$ is under consideration, the possible failure time t is assumed to be contained in $[t_{mn}, t_{mx}]$; further, if a link has not failed within $[t_{mn}, t_{mx}]$, its failure time is set to a value greater than t_{mx} for use with the indicator functions $\delta_i(t \mid \mathbf{t})$ defined in Eqs. (A.15)-(A.18).

The expected value $E[\delta_i(t\,|\,\mathbf{t})]$, i = 1, 2, 3, 4, for $\delta_i(t\,|\,\mathbf{t})$ corresponds to the PLOAS value $pF_i(t)$ defined in Table A.1. Approach MC1 uses random sampling from the CDFs $CDF_{WL,j}(\tau)$, j = 1, 2, …, nWL, and $CDF_{SL,j}(\tau)$, j = 1, 2, …, nSL, for link failure times in the estimation of PLOAS values. In this approach, $pF_i(t)$ is approximated by

$$\begin{aligned} pF_i(t) &= \int_{I^{nL}} \delta_i\left[t\,|\,\mathbf{f}(\mathbf{r})\right]\prod_{k=1}^{nL} d_k(r_k)\prod_{k=1}^{nL} \mathrm{d}r_k \\ &= \int_{I^{nL}} \delta_i\left[t\,|\,\mathbf{f}(\mathbf{r})\right]\prod_{k=1}^{nL} \mathrm{d}r_k \\ &\cong \sum_{l=1}^{nR} \delta_i\left[t\,|\,\mathbf{f}(\mathbf{r}_l)\right]\Big/ nR \\ &= \sum_{l=1}^{nR} \delta_i\left\{t\,|\left[tWL_{1l}, tWL_{2l}, \ldots, tWL_{nWL,l}, tSL_{1l}, tSL_{2l}, \ldots, tSL_{nSL,l}\right]\right\}\Big/ nR, \end{aligned} \tag{A.19}$$

where (i) $nL = nWL + nSL$, (ii) $d_k(r_k) = 1$ is the density function for a variable r_k with a uniform distribution on [0, 1], (iii) $I^{nL} = [0,1]^{nL}$ (i.e., the unit cube of dimension nL), (iv) $\mathbf{r} = [r_1, r_2, \ldots, r_{nL}] \in I^{nL}$, (v) the function $\mathbf{f}(\mathbf{r})$ is defined by

$$\begin{aligned} \mathbf{f}(\mathbf{r}) &= \Big[CDF_{WL,1}^{-1}(r_1), CDF_{WL,2}^{-1}(r_2), \ldots, CDF_{WL,nWL}^{-1}(r_{nWL}), \\ &\qquad CDF_{SL,1}^{-1}(r_{nWL+1}), CDF_{SL,2}^{-1}(r_{nWL+2}), \ldots, CDF_{SL,nSL}^{-1}(r_{nL})\Big] \\ &= \left[tWL_1, tWL_2, \ldots, tWL_{nWL}, tSL_1, tSL_2, \ldots, tSL_{nSL}\right] \end{aligned} \tag{A.20}$$

with $tWL_j = CDF_{WL,j}^{-1}(r_j)$ for j = 1, 2, …, nWL and $tSL_j = CDF_{SL,j}^{-1}(r_{nWL+j})$ for j = 1, 2, …, nSL, and (vi) $\mathbf{r}_l$, l = 1, 2, …, nR, is a random sample of size nR from a uniform distribution on I^{nL}. With respect to the approximation of $pF_i(t)$ in Eq. (A.19), the first equality defines $pF_i(t)$ as the expected value of $\delta_i[t\,|\,\mathbf{f}(\mathbf{r})]$; the second equality is a notational simplification based on the equalities $d_k(r_k) = 1$ for k = 1, 2, …, nL; the approximation at the third step is based on a random sample from the link failure times; and the final equality is a restatement of $\mathbf{f}(\mathbf{r})$ in terms of link failure times.

For computational efficiency, CPLOAS_2 implements MC1 as a two-step procedure. In the first step, *nFT* vectors of link failure times of the form indicated in Eq. (A.20) are generated. In the second step, *nFTC* vectors of link failure times of the form indicated in Eq. (A.20) are generated by randomly sampling from the link failure times generated in Step 1 (i.e., from the *nFT* failure times for each link). Then, the *nFTC* vectors of link failure times generated in Step 2 are used in Eq. (A.19) in the estimation of PLOAS. For this approach to be effective, *nFTC* must be significantly larger than *nFT*.

In addition, CPLOAS_2 also determines a variance and standard error for the estimate $\widehat{pF}_i(t)$ for $pF_i(t)$ in Eq. (A.19). Specifically, it follows from the Central Limit Theorem that

$$\frac{\widehat{pF}_i(t) - pF_i(t)}{s_i(t)\big/\sqrt{nR}} \tag{A.21}$$

is approximately distributed as a normal distribution with mean 0 and variance 1 (i.e., is $N(0, 1)$), where

$$s_i(t) = \left[\sum_{l=1}^{nR}\left(\delta_i\left\{t \mid \left[tWL_{1l}, tWL_{2l}, \ldots, tWL_{nWL,l}, tSL_{1l}, tSL_{2l}, \ldots, tSL_{nSL,l}\right]\right\} - \widehat{pF}_i(t)\right)^2 \Big/ nR\right]^{1/2} \tag{A.22}$$

and nR is sufficiently large ([4], p. 75). In turn, the quantity $s_i / \sqrt{nR}$ can be used to assess the potential error in the approximation $\widehat{pF}_i(t)$ for $pF_i(t)$ in Eq. (A.19). More specifically, if $z_{\alpha/2}$ is the $1-\alpha/2$ quantile of the unit normal distribution $N(0, 1)$, then

$$prob\left(\widehat{pF}_i(t) - z_{\alpha/2}s_i(t)\big/\sqrt{nR} \le pF_i(t) \le \widehat{pF}_i(t) + z_{\alpha/2}s_i(t)\big/\sqrt{nR}\right) = 1-\alpha \tag{A.23}$$

where $prob(\sim)$ denotes probability ([5], pp. 168-169). In turn,

$$\left[\widehat{pF}_i(t) - z_{\alpha/2}s_i(t)\big/\sqrt{nR}, \widehat{pF}_i(t) + z_{\alpha/2}s_i(t)\big/\sqrt{nR}\right] \tag{A.24}$$

is a $100(1-\alpha)$ percent confidence interval for $pF_i(t)$. As an example, $z_{\alpha/2}$ = 1.96 for a 95% confidence interval. Because the sample size nR used in the estimation of $pF_i(t)$ in Eq. (A.19) will be a large integer, it is acceptable to use the unit normal distribution in the estimation of the confidence interval in Eq. (A.24) rather than the *t*-distribution, which would be used if *nR* was a very small integer.

For completeness, the computational implementation of the two-step sampling procedure used in CPLOAS_2 for MC1 is now described in more detail. In the first step, failure times for the individual links are randomly sampled *nFT* times from the CDFs for link failure time. This produces sets

$$\mathcal{WL}_j = \left\{tWL_{jl} : l = 1, 2, \ldots, nFT\right\},\ j = 1, 2, \ldots, nWL \tag{A.25}$$

and

$$\mathcal{SL}_j = \left\{tSL_{jl} : l = 1,2,...,nFT\right\}, \; j = 1,2,...,nSL \tag{A.26}$$

of *nFT* randomly-sampled link failure times for each link, where (i) tWL_{jl}, l = 1, 2, …, *nFT*, are the sampled failure times for WL *j*, and (ii) tSL_{jl}, l = 1, 2, …, *nFT*, are the sampled failure times for SL *j*.

In the second step, failure times are randomly sampled *nFTC* times with replacement from the sets $\mathcal{WL}_j$ and $\mathcal{SL}_j$ to produce the following sequence of vectors

$$\mathbf{t}_l = \left[tWL_{1l}, tWL_{2l}, ..., tWL_{nWL,l}, tSL_{1l}, tSL_{2l}, ..., tSL_{nSL,l}\right], l = 1,2,...,nFTC, \tag{A.27}$$

of possible link failure times in Eq. (A.19) with *nFTC* corresponding to the sample size *nR* in Eq. (A.19). In turn, each vector $\mathbf{t}_l$ of link failure times results in a corresponding time tF_{il} at which LOAS occurs obtained in consistency with the definition of LOAS under consideration (i.e., i = 1, 2, 3, 4 as indicated in Eqs. (A.15)-(A.18)), which is the failure time used in the evaluation of the function $\delta_i[t \mid \mathbf{t}_l]$ in the final equality of Eq. (A.19).

Specifically, $pF_i(t)$ is approximated in CPLOAS_2 by

$$\widehat{pF_i}(t) = \sum_{l=1}^{nFTC} \delta(t \mid tF_{il}) / nFTC \;\; \text{with} \;\; \delta(t \mid tF_{il}) = \begin{cases} 1 & \text{if } tF_{il} \le t \\ 0 & \text{otherwise.} \end{cases} \tag{A.28}$$

The computational procedure implemented within CPLOAS_2 does not save the times tF_{il} until the end of the calculation and then determine $\widehat{pF_i}(t)$ as indicated in Eq. (A.28). Rather, a running sum

$$S_{i0}(t) = 0, \quad S_{il}(t) = S_{i,l-1}(t) + \delta(t \mid tF_{il}), l = 1,2,...,nFTC, \tag{A.29}$$

of the functions $\delta(t \mid tF_{il})$ is performed that yields

$$\widehat{pF_i}(t) = S_{i,nFTC}(t) / nFTC \tag{A.30}$$

at the end of the calculation but does not save the individual times tF_{il}.

The standard deviation associated with the estimate for $pF_i(t)$ in Eq. (A.28) is given by

$$
\begin{aligned}
s_i(t) &= \left\{ \sum_{l=1}^{nFTC} \left[\delta(t \mid tF_{il}) - \widehat{pF_i}(t) \right]^2 \Big/ nFTC \right\}^{1/2} \Big/ \sqrt{nFTC} \\
&= \left\{ \sum_{l=1}^{nFTC} \left[\delta^2(t \mid tF_{il}) \big/ nFTC \right] - \widehat{pF_i}^{\,2}(t) \right\}^{1/2} \Big/ \sqrt{nFTC} \\
&= \left\{ \left[S_{i,nFTC}(t) \big/ nFTC \right] - \widehat{pF_i}^{\,2}(t) \right\}^{1/2} \Big/ \sqrt{nFTC} \\
&= \left\{ \widehat{pF_i}(t) - \widehat{pF_i}^{\,2}(t) \right\}^{1/2} \Big/ \sqrt{nFTC}
\end{aligned}
\tag{A.31}
$$

with the problem reformulation associated with the third equality possible because $\delta(t \mid tF_{il})$ is always either 0 or 1.

With respect to the two step sampling in use, (i) the first sample from the link failure CDFs is, in effect, a numerical procedure to facilitate the evaluation of the link failure CDFs, and (ii) the second sample corresponds to the sample used in the evaluation of the integral in Eq. (A.19).

A.4 Sampling Procedure 2 (MC2)

Sampling procedure 2 (MC2) is similar to sampling procedure 1 (MC1) but with use of the distributions for the variables $\alpha_{WL,j}, \beta_{WL,j}$, j = 1, 2, …, nWL, and $\alpha_{SL,j}, \beta_{SL,j}$, j = 1, 2, …, nSL, indicated in conjunction with Eqs. (A.1)-(A.6) that define properties and failure values for the individual links. The approximation to $pF_i(t)$ for MC2 is analogous to the approximation in Eq. (A.19) for MC1 but with changed definitions for $\mathbf{r}$, $\mathbf{r}_l$, $\mathbf{f}(\mathbf{r})$ and $d_k(r_k)$. Specifically,

$$
\begin{aligned}
\mathbf{r} &= \left[r_1, r_2, \ldots, r_{2nL} \right] \text{ with } nL = nWL + nSL \\
&= \left[\alpha_{WL,1}, \alpha_{WL,2}, \ldots, \alpha_{WL,nWL}, \beta_{WL,1}, \beta_{WL,2}, \ldots, \beta_{WL,nWL}, \right. \\
&\qquad \left. \alpha_{SL,1}, \alpha_{SL,2}, \ldots, \alpha_{SL,nSL}, \beta_{SL,1}, \beta_{SL,2}, \ldots, \beta_{SL,nSL} \right],
\end{aligned}
\tag{A.32}
$$

$$
\mathbf{p}_{WL,j} = \left[\alpha_{WL,j}, \beta_{WL,j} \right] = \left[r_j, r_{nWL+j} \right], \; j = 1, 2, \ldots, nWL, \tag{A.33}
$$

$$
\mathbf{p}_{SL,j} = \left[\alpha_{SL,j}, \beta_{SL,j} \right] = \left[r_{2nWL+j}, r_{2nWL+nSL+j} \right], \; j = 1, 2, \ldots, nSL, \tag{A.34}
$$

$$
\begin{aligned}
f_{WL,j}\left(\mathbf{p}_{WL,j}\right) &= \text{ time at which WL } j \text{ fails with } \mathbf{p}_{WL,j} = \left[\alpha_{WL,j}, \beta_{WL,j} \right] \\
&= tWL_j,
\end{aligned}
\tag{A.35}
$$

$$
\begin{aligned}
f_{SL,j}\left(\mathbf{p}_{SL,j}\right) &= \text{ time at which SL } j \text{ fails with } \mathbf{p}_{SL,j} = \left[\alpha_{SL,j}, \beta_{SL,j}\right] \\
&= tSL_j,
\end{aligned}
\tag{A.36}
$$

and

$$
\begin{aligned}
\mathbf{f}(\mathbf{r}) = \Big[& f_{WL,1}\left(\mathbf{p}_{WL,1}\right), f_{WL,2}\left(\mathbf{p}_{WL,2}\right), \ldots, f_{WL,nWL}\left(\mathbf{p}_{WL,nWL}\right) \\
& f_{SL,1}\left(\mathbf{p}_{SL,1}\right), f_{SL,2}\left(\mathbf{p}_{SL,2}\right), \ldots, f_{SL,nSL}\left(\mathbf{p}_{WL,nSL}\right)\Big] \\
= \big[& tWL_1, tWL_2, \ldots, tWL_{nWL}, tSL_1, tSL_2, \ldots, tSL_{nSL}\big].
\end{aligned}
\tag{A.37}
$$

In turn,

$$
\begin{aligned}
pF_i(t) &= \int_{\mathcal{S}} \delta_i\left[t \mid \mathbf{f}(\mathbf{r})\right] \prod_{k=1}^{2nL} d_k\left(r_k\right) \prod_{k=1}^{2nL} \mathrm{d}r_k \\
&\cong \sum_{l=1}^{nR} \delta_i\left[t \mid \mathbf{f}\left(\mathbf{r}_l\right)\right] \Big/ nR \\
&= \sum_{l=1}^{nR} \delta_i\left\{t \mid \left[tWL_{1l}, tWL_{2l}, \ldots, tWL_{nWL,l}, tSL_{1l}, tSL_{2l}, \ldots, tSL_{nSL,l}\right]\right\} \Big/ nR,
\end{aligned}
\tag{A.38}
$$

where (i) $d_k(r_k)$ is the density function for r_k defined on the set $\mathcal{S}_k$ of possible values for r_k (i.e., for $\alpha_{WL,j}$, $\beta_{WL,j}$, $\alpha_{SL,j}$ or $\beta_{SL,j}$ as appropriate; see Eq. (A.32)), (ii) $\mathcal{S} = \mathcal{S}_1 \times \mathcal{S}_2 \times \cdots \times \mathcal{S}_{2nL}$, and (iii) $\mathbf{r}_l$, l = 1, 2, …, nR, is a random sample of size nR from $\mathcal{S}$ generated in consistency with the distributions defined by the density functions $d_k(r_k)$.

Unlike MC1, CPLOAS_2 implements MC2 with a single sampling step. Specifically, *nFTC* vectors of the form indicated in Eq. (A.32) are randomly generated and then used in the indicated sequence of calculations that lead to the approximation $\widehat{pF}_i(t)$ for $pF_i(t)$ in Eq. (A.38). As for MC1, confidence intervals for the approximation $\widehat{pF}_i(t)$ for $pF_i(t)$ obtained for MC2 are calculated as indicated in Eqs. (A.21)-(A.24).

A.5 Importance Sampling Procedure 1 (IMP1)

Importance sampling procedure 1 (IMP1) involves the use of importance sampling in the evaluation of the integral in Eq. (A.19) for the problem formulation described for MC1 (i.e., with sampling from the link failure time CDFs). Because the failure of SLs is less likely than the failure of WLs, the importance sampling procedure implemented in CPLOAS_2 for the evaluation of the integral in Eq. (A.19) uses right triangular importance sampling distributions for WLs and left triangular importance sampling distributions for SLs, which results in an overemphasis for large WL failure times and small SL failure times. Further, the importance sampling is performed on the cumulative probabilities associated with the link CDFs rather than directly on the link failure times. This choice was made as the calculation of the link CDFs in CPLOAS_2 made these probabilities available. In contrast, importance sampling directly on the link failure times would have required the use of density functions for link time, which were not calculated.

Because the cumulative probabilities associated link failure probabilities are being sampled, the indicated right and left triangular importance sampling distributions $d_{IR}(r)$ and $d_{IL}(r)$ are defined on the interval [0, 1] by

$$d_{IR}(r) = 2r \text{ and } d_{IL}(r) = 2 - 2r \text{ for } 0 \le r \le 1. \tag{A.39}$$

Introduction of the importance sampling distributions $d_{IR}(r)$ and $d_{IL}(r)$ into Eq. (A.19) results in the following approximation for $pF_i(t)$:

$$\begin{aligned} pF_i(t) &= \int_{I^{nL}} \left\{ \delta_i\left[t \mid \mathbf{f}(\mathbf{r})\right] \Big/ \prod_{k=1}^{nL} d_{I,k}\left(r_k\right) \right\} \prod_{k=1}^{nL} d_{I,k}\left(r_k\right) \prod_{k=1}^{nL} \mathrm{d}r_k \\ &\cong \sum_{l=1}^{nR} \left\{ \delta_i\left[t \mid \mathbf{f}\left(\mathbf{r}_l\right)\right] \Big/ \prod_{k=1}^{nL} d_{I,k}\left(r_{kl}\right) \right\} / nR \\ &= \sum_{l=1}^{nR} \frac{\delta_i\left\{t \mid \left[tWL_{1l}, tWL_{2l}, \ldots, tWL_{nWL,l}, tSL_{1l}, tSL_{2l}, \ldots, tSL_{nSL,l}\right]\right\}}{nR \prod_{k=1}^{nL} d_{I,k}\left(r_{kl}\right)}, \end{aligned} \tag{A.40}$$

where (i) the first equality derives from the introduction of the importance sampling distributions defined by the right and left density functions $d_{I,k}(r_k)$, k = 1, 2, …, nL, for WLs and SLs into the representation for $pF_i(t)$ in Eq. (A.19), (ii) the following approximation involves a random sample $\mathbf{r}_l$, l = 1, 2, …, nR, from I^{nL} generated in consistency with the distributions defined by the density functions $d_{I,k}(r_k)$, and (iii) the final equality is a restatement of $\mathbf{f}(\mathbf{r})$ in terms of link failure times.

Rather than use the result in Eq. (A.40) as the final approximation for $pF_i(t)$, IMP1 is implemented with a two-step sampling procedure that is analogous to the two-step sampling procedure used for MC1. The importance sampling that generates the link failure times in Eq. (A.40) corresponds to the first step in the two-step sampling procedure for IMP1. The second step in the two-step sampling procedure for IMP1 involves a sampling of the link failure times generated in the first step. For consistency in terminology with MC1, the first and second samplings of link failure time for IMP1 are referred to as being of size *nFT* and *nFTC*, respectively. As for MC1, this approach to the determination of IMP1 is only effective if *nFTC* is much larger than *nFTC*. If *nFTC* is not much larger than *nFTC*, then use of the initial approximation to $pF_i(t)$ in Eq. (A.40) will most likely be better than the result of approximation procedure that is now described.

The sampling procedure that generates the approximation in Eq. (A.40) corresponds to the first step in IMP1and produces the following set of results for $nR = nFT$:

$$\mathcal{L}_k = \{[tL_{kl}, wL_{kl}], l = 1, 2, ..., nFT\}, k = 1, 2, ..., nL, \tag{A.41}$$

where (i)

$$\mathbf{r}_l = [r_{1l}, r_{2l}, ..., r_{nL,l}], l = 1, 2, ..., nFT, \tag{A.42}$$

is a sample from I^{nL} generated in consistency with the importance sampling distributions defined by $d_{Ik}(r)$ for the individual links (i.e., r_{kl} is a random sample from [0, 1] generated in consistency with the importance sampling distribution defined by $d_{Ik}(r)$ for link k; see Eq. (A.39)), (ii)

$$tL_{kl} = \begin{cases} tWL_{kl} & \text{for } k = 1, 2, ..., nWL \\ tSL_{k-nWL,l} & \text{for } k = nWL+1, nWL+2, ..., nL \end{cases} \tag{A.43}$$

is the failure time for link k obtained with element r_{kl} of $\mathbf{r}_l$, and (iii)

$$wL_{kl} = 1 \big/ \left[nFT \ d_{Ik}\left(r_{kl}\right)\right] \tag{A.44}$$

is the importance sampling weight associated with the sampled link failure time tL_{kl}.

In concept, $pF_i(t)$ can be approximated by consideration of all possible combinations of the link failure times associated with the sets $\mathcal{L}_k$, k = 1, 2,…, nL, in Eq. (A.41) as indicated in the following summation:

$$pF_i(t) \cong \sum_{\mathbf{s} \in \mathcal{S}} \delta_i[t \mid \mathbf{t}(\mathbf{s})] w[\mathbf{t}(\mathbf{s})] \tag{A.45}$$

where

$$\mathcal{S} = \left\{\mathbf{s} : \mathbf{s} = [s(1), s(2), ..., s(nL)] \in \prod_{i=1}^{nL} \{1, 2, ..., nL\}_i\right\}, \tag{A.46}$$

$$\mathbf{t}(\mathbf{s}) = \left[tL_{1,s(1)}, tL_{2,s(2)}, ..., tL_{nL,s(nL)}\right], \tag{A.47}$$

$$\begin{aligned} w[\mathbf{t}(\mathbf{s})] &= \prod_{k=1}^{nL} wL_{k,s(k)} \\ &= \prod_{k=1}^{nL} 1 \big/ \left[nFT \ d_{Ik}\left(r_{k,s(k)}\right)\right] \\ &= 1 \big/ \left[nFT^{nL} \prod_{k=1}^{nL} d_{Ik}\left(r_{k,s(k)}\right)\right], \end{aligned} \tag{A.48}$$

and the summation in Eq. (A.45) involves nFT^{nL} terms (i.e., the number of elements in the set $\mathcal{S}$).

For IMP1, the summation in Eq. (A.45) is approximated by sampling from $\mathcal{S}$ with each element $\mathbf{s}$ of $\mathcal{S}$ assigned a probability of $1/nFT^{nL} = nFT^{-nL}$.This produces the following approximation to $pF_i(t)$:

$$\begin{aligned} pF_i(t) &\cong \sum_{\mathbf{s}\in\mathcal{S}} \left\{ \frac{\delta_i\left[t\,|\,\mathbf{t}(\mathbf{s})\right] w\left[\mathbf{t}(\mathbf{s})\right]}{1/nFT^{nL}} \right\} \left\{ 1/nFT^{nL} \right\} \\ &= \sum_{\mathbf{s}\in\mathcal{S}} \left[\frac{\delta_i\left[t\,|\,\mathbf{t}(\mathbf{s})\right]}{nFT^{nL} \prod_{k=1}^{nL} d_{Ik}\left(r_{k,s(k)}\right) / nFT^{nL}} \right] \left[\frac{1}{nFT^{nL}} \right] \\ &= \sum_{\mathbf{s}\in\mathcal{S}} \left[\frac{\delta_i\left[t\,|\,\mathbf{t}(\mathbf{s})\right]}{\prod_{k=1}^{nL} d_{Ik}\left(r_{k,s(k)}\right)} \right] \left[\frac{1}{nFT^{nL}} \right] \\ &\cong \sum_{l=1}^{nFTC} \left[\frac{\delta_i\left[t\,|\,\mathbf{t}(\mathbf{s}_l)\right]}{\prod_{k=1}^{nL} d_{Ik}\left(r_{k,s(k,l)}\right)} \right] \left[\frac{1}{nFTC} \right], \end{aligned} \tag{A.49}$$

where

$$\mathbf{s}_l = \left[s(1,l), s(2,l), ..., s(nL,l) \right], l = 1,2,...,nFTC, \tag{A.50}$$

is a uniform random sample of size $nFTC$ from $\mathcal{S}$.

The final approximation $\widehat{pF}_i(t)$ for $pF_i(t)$ in Eq. (A.49) completes the second step of the two-step importance sampling procedure used for IMP1 in CPLOAS_2.

In addition, the standard deviation for $\widehat{pF}_i(t)$ is given by

$$\begin{aligned} s_i &= \left\{ \sum_{l=1}^{nFTC} \left[\frac{\delta_i\left[t\,|\,\mathbf{t}(\mathbf{s}_l)\right]}{\prod_{k=1}^{nL} d_{Ik}\left(r_{k,s(k,l)}\right)} - \widehat{pF}_i(t) \right]^2 \left[\frac{1}{nFTC} \right] \right\}^{1/2} \Bigg/ \sqrt{nFTC} \\ &= \left\{ \sum_{l=1}^{nFTC} \left[\frac{\delta_i\left[t\,|\,\mathbf{t}(\mathbf{s}_l)\right]}{\prod_{k=1}^{nL} d_{Ik}\left(r_{k,s(k,l)}\right)} \right]^2 \left[\frac{1}{nFTC} \right] - \widehat{pF}_i^{\,2}(t) \right\}^{1/2} \Bigg/ \sqrt{nFTC}. \end{aligned} \tag{A.51}$$

In turn, s_i is used in the determination of confidence intervals for the approximation $\widehat{pF}_i(t)$ for $pF_i(t)$ obtained with IMP1 as indicated in Eqs. (A.21)-(A.24). Similarly to the procedure described in conjunction with Eqs. (A.28)-(A.30) for MC1, running sums are used in CPLOAS_2 in the calculation of $\widehat{pF}_i(t)$ and s_i.

C.6 Importance Sampling Procedure 2 (IMP2)

Importance sampling procedure 2 (IMP2) involves the use of importance sampling in the evaluation of the integral in Eq. (A.19) for the problem formulation described for MC2 (i.e., with sampling from the α's and β's for the individual links). Because the failure of SLs is less likely than the failure of WLs, the importance sampling procedure implemented in CPLOAS_2 for the evaluation of the integral in Eq. (A.19) uses right triangular importance sampling distributions for WL β's and SL α's and left triangular importance sampling distributions for WL α's and SL β's, which results in an overemphasis for large WL failure times and small SL failure times. Further, the importance sampling is performed on the cumulative probabilities associated with the α's and β's rather than directly on the α's and β's.

Introduction of the importance sampling distributions $d_{IR}(r)$ and $d_{IL}(r)$ defined in Eq. (A.39) into Eq. (A.19) results in the following approximations for $pF_i(t)$:

$$
\begin{aligned}
pF_i(t) &= \int_{I^{2nL}} \left\{ \delta_i\left[t \mid \mathbf{f}(\mathbf{r})\right] \Big/ \prod_{k=1}^{2nL} d_{I,k}(r_k) \right\} \prod_{k=1}^{2nL} d_{I,k}(r_k) \prod_{k=1}^{2nL} \mathrm{d}r_k \\
&\cong_1 \sum_{l=1}^{nR} \left\{ \delta_i\left[t \mid \mathbf{f}(\mathbf{r}_l)\right] \Big/ \prod_{k=1}^{2nL} d_{I,k}(r_{kl}) \right\} / \, nR \\
&= \sum_{l=1}^{nR} \left[\frac{\delta_i\left\{t \mid \left[tWL_{1l}, tWL_{2l}, \ldots, tWL_{nWL,l}, tSL_{1l}, tSL_{2l}, \ldots, tSL_{nSL,l}\right]\right\}}{\prod_{k=1}^{2nL} d_{I,k}(r_{kl})} \right] / \, nR \\
&\cong_2 \sum_{l=1}^{nR} \left[\frac{\delta_i\left\{t \mid \left[tWL_{1l}, tWL_{2l}, \ldots, tWL_{nWL,l}, tSL_{1l}, tSL_{2l}, \ldots, tSL_{nSL,l}\right]\right\}}{\prod_{k=1}^{2nL} d_{I,k}(r_{kl})} \right] \Bigg/ \sum_{l=1}^{nR} \prod_{k=1}^{2nL} d_{I,k}(r_{kl}) \\
&= \sum_{l=1}^{nR} \left\{ \delta_i\left[t \mid \mathbf{f}(\mathbf{r}_l)\right] \Big/ \prod_{k=1}^{2nL} d_{I,k}(r_{kl}) \right\} \Bigg/ \sum_{l=1}^{nR} \prod_{k=1}^{2nL} d_{I,k}(r_{kl}),
\end{aligned}
\tag{A.52}
$$

where (i) the first equality derives from the introduction of the importance sampling distributions defined by the right and left density functions $d_{I,k}(r_k)$, k = 1, 2, …, nL, for the α's and β's for the individual links into the representation for $pF_i(t)$ in Eq. (A.19), (ii) the following approximation (i.e., $\cong_1$) involves a random sample $\mathbf{r}_l$, l = 1, 2, …, nR, from I^{2nL} generated in consistency with the distributions defined by the density functions $d_{I,k}(r_k)$ with a sampling weight for each observation equal to the reciprocal of the sample size (i.e., $1/nR$) , (iii) the next equality is a restatement of $\mathbf{f}(\mathbf{r})$ in terms of link failure times as indicated in Eqs. (A.32)-(A.37), (iv) the second approximation (i.e., $\cong_2$) results from replacing the reciprocal of the sample size (i.e., $1/nR$) with the reciprocal of the sum weights from the importance sampling, and (v) the final equality results from a return to the previously used more compact representation for the indicator function for LOAs (i.e., $\delta_i[t \mid \mathbf{f}(\mathbf{r}_l)]$). For convenience, the first and second approximation procedures will be referred to as IMP2_1 and IMP2_2, respectively. The replacement of $1/nR$ in the definition of IMP2_1 by

$$1\Big/\sum_{l=1}^{nR}\prod_{k=1}^{2nL} d_{I,k}\left(r_{kl}\right) \tag{A.53}$$

in the definition of IMP2_2 is suggested by some authors (e.g., Refs. [6; 7]).

The use of IMP2_1 or IMP2_2 is possible in CPLOAS_2 as a user-specified option. At present, IMP2_1 is the recommended option for use in the implementation of IMP2 as there is currently limited experience with the use of IMP2_2 as an option for the implementation of IMP2. For consistency, CPLOAS_2 uses the same sample size *nFTC* for IMP2_1 and IMP2_2 as used for MC1, MC2 and IMP1; thus, the indicated sample size *nR* in Eq. (A.52) corresponds to the sample size *nFTC* in the notation used in CPLOAS_2. As for MC2, a single sample of size *nFTC* is used in CPLOAS_2 for IMP2_1 and IMP2_2.

For IMP2_1, the standard deviation for $\widehat{pF}_i(t)$ is given by

$$\begin{aligned} s_i &= \left\{\sum_{l=1}^{nFTC}\left\{\delta_i\left[t \mid \mathbf{f}\left(\mathbf{r}_l\right)\right]\Big/\prod_{k=1}^{2nL} d_{I,k}\left(r_{kl}\right) - \widehat{pF}_i(t)\right\}^2 / \, nFTC\right\}^{1/2} \Bigg/ \sqrt{nFTC} \\ &= \left\{\sum_{l=1}^{nFTC}\left\{\delta_i\left[t \mid \mathbf{f}\left(\mathbf{r}_l\right)\right]\Big/\prod_{k=1}^{2nL} d_{I,k}\left(r_{kl}\right)\right\}^2 \Big/ nFTC - \widehat{pF}_i^{\,2}(t)\right\}^{1/2} \Bigg/ \sqrt{nFTC}\,. \end{aligned} \tag{A.54}$$

As suggested in Refs. [6; 7], the standard deviation for $\widehat{pF}_i(t)$ obtained for IMP2_2 is defined in CPLOAS_2 by

$$\begin{aligned} s_i &= \left\{\sum_{l=1}^{nFTC} w^2\left(\mathbf{r}_l\right)\left\{\delta_i\left[t \mid \mathbf{f}\left(\mathbf{r}_l\right)\right] - \widehat{pF}_i(t)\right\}^2 \Big/ \left[\sum_{l=1}^{nFTC} w\left(\mathbf{r}_l\right)\right]^2\right\}^{1/2} \\ &= \frac{\left(\sum_{l=1}^{nFTC} w^2\left(\mathbf{r}_l\right)\left\{\delta_i\left[t \mid \mathbf{f}\left(\mathbf{r}_l\right)\right] - \widehat{pF}_i(t)\right\}^2\right)^{1/2}}{\sum_{l=1}^{nFTC} w\left(\mathbf{r}_l\right)} \end{aligned} \tag{A.55}$$

with

$$w(\mathbf{r}_l) = 1 \Big/ \prod_{k=1}^{2nL} d_{I,k}(r_{kl}). \qquad \text{(A.56)}$$

For both IMP2_1 and IMP2_2, CPLOAS_2 uses the indicated values for s_i to determine confidence intervals for $\widehat{pF}_i(t)$ as described in conjunction with Eqs. (A.21)-(A.24).

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